Viewing the world systemically.

Axiomatic Theory of Intentional Systems (*ATIS*), and Options-Set Analyses for Education

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Abstract

Axiomatic Theory of Intentional Systems (*ATIS*) is a scientific theory that provides the means to evaluate learning and any other intentional-systems environments without having to conduct extensive empirical hypothesis-based research, thus saving time and money, and possibly years of waiting to determine whether a proposed change to a learning environment actually results in the desired outcomes.

The *Options-Set* is simply *Relation-Set* and *Object-Set* that generates the *System-Properties*, and thereby the *Axiom-Set* that generates the theorems of the theory; that is, the outcomes of the system.

The objective of this theory is to provide the means to obtain an individual theory for each application of an *Options-Set*, thus making it possible to obtain actual individual results for specific schools rather than just a statistical probability that a proposed change may obtain the desired results—*ATIS* provides for scientific certainty not possible with hypothesis-driven research.

ATIS is an emendation of SIGGS that was developed by Steiner and Maccia (1966), and incorporates research, in particular, by the biologist Ludwig

von Bertalanffy (1972), the engineer and mathematician Mihajlo D. Mesarović (1972), and the mathematician Yi Lin (1999).

ATIS has been developed by means of the logical process of retroduction whereby one theory, SIGGS, is used as a model for developing another theory, *ATIS*.

The results of utilizing *ATIS* to predict education outcomes of individual school systems can be visualized by the Power Point demonstration of just such an application to a school system, *Predicting Outcomes of Systemic Change in Education*, an analysis that demonstrates the application of axioms and theorems for determining education outcomes.

The demonstration can be seen at:

https://www.indiana.edu/~tedfrick/aect2006/POSCEaect2006.ppt.

Further, the dynamic properties of a general system as defined by ATIS can be visualized by the demonstration seen at the following link:

http://educology.indiana.edu/Thompson/ATIStutorials.html

Introduction

This report provides developments of Thompson's *Axiomatic Theory of Intentional Systems* (*ATIS*) and *ATIS*-Analyses as they provide predictive outcomes for education systems. *ATIS*, founded on an emendation of *General Systems Theory*, provides the means to predict events individually, rather than having to wait for "patterns" to develop as is required with data-mining analyses. In this way we can predict; for example, what would happen in an education system if certain modifications are implemented without having to wait for 12 years or more to see if the changes will actually result in the desired outcomes. Further an *ATIS*-topological-analysis provides the means to effectively integrate and use metadata in a manner that provides real-time results and yet maintains personal security that is of concern to everyone in a free society. This development is discussed since it is critical to being able to implement changes in a system immediately and know immediately what the outcomes will be if the changes are implemented. While the full value of topological analyses have not been developed in this report, the properties of a topology that should be of value are presented, as well as a constructive decision-procedure for determining a topology for any intentional system is given.

Of significant importance to education theorists is the development of a means to convert hypothesis-driven research to an axiomatic theory that has real predictive outcomes for education systems. The value of an axiomatic analysis was shown above by PESO and the Adobe Flash Player demonstration of ATIS applications that shows how axioms help to predict what will happen when certain changes are made in an education system.

After viewing those presentations, consider the following application of *ATIS* to an actual problem encountered in a school system in Oregon, and see how *ATIS* could have provided a solution to their problems.

An *ATIS*-Analysis for Decision-Making: Successful Implementation of a "Small Schools" Project

From a report concerning the implementation of a "Small Schools" project in Lebanon, Oregon (see article at the following link: Small Schools Project, Lebanon, Oregon); it appears as though there is a disjunct between the plan and its implementation leading to dissatisfaction with the plan, when in fact the problems appear to be mainly in the implementation of the plan.

From economic theories' economies of size, it is known that organizationally there are savings that generally accrue due to control by large school systems.

From pedagogic theories it is known that intimate organizations, not necessarily small, result in better academic outcomes.

Both of these theories are confirmed by *ATIS* wherein economies of size are the result of the system property ***wholeness*** and academic outcomes are the result of the system property ***interdependentness***.

From *ATIS*, we have the following axioms:

70. If ***wholeness*** increases and hierarchical order is constant, then integration increases.

63. If interdependence increases, then complexity growth increases.

That is, from Axiom 70, by maintaining the normal *hierarchical structure* of the school system while increasing the *wholeness* of the system; that is, the number of affect relations, then *integration* increases. In the case of *economic wholeness*, *economic integration* means that there is more *economic control*. Essentially, from Axiom 70, we see that the larger the system, the greater the economic efficiency.

From Axiom 63, we see that as *classroom interdependence* increases, the greater *classroom complexity growth* is realized. Where the *classroom interdependence* is related to learning activities, the greater the learning is that results. When this *classroom interdependence* can be best realized in smaller classes, then the smaller classes will result in greater learning.

The goal, therefore, is to obtain the economic returns that accrue from economy of size resulting from large school systems, while applying an intimate organizational structure that realizes the close relationships required for individual student academic success.

This is where *ATIS* and its applications developed by Theodore W. Frick, can be most valuable. See Professor Frick's analysis at: https://www.indiana.edu/~aptfrick/overview/

Predicting School Outcomes Before Expending Thousands of Dollars—The Systemic Approach to Change

The problems in Lebanon, Oregon, highlight the problems of restructuring school systems throughout the United States. We will consider the problems reported there and the alternative solutions that would have realized a better outcome for the school system as an example of what *ATIS* and its applications can do to help solve these problems **before** expending thousands of dollars on a project where it is known that it has little chance of success before it starts.

The "Small Schools" concept is clearly stated in the Lebanon report:

> The idea behind small schools conversion is simple: Students in large, anonymous high schools are separated into groups of about 300 apiece, often according to academic interest, and go through high school together, with the same group of teachers for all four years. The idea is for teachers to get to know students beyond just a name that disappears after nine months slouching at the back of the classroom.

However, the implementation of this vision was that the school actually had to be separated into four different buildings. The economic results should have been clear from the beginning as subsequently recognized by Rick Alexander:

> School board chair Rick Alexander said he had long been concerned about the higher administrative costs that small schools bring, since each of the schools-within-a-school has its own principal, and with replication of core curriculum courses in each of the four academies.

What is needed, however, is a *large-school structure with a small-school feel*. This is what is meant by the *intimate organization regardless of organization size*. There is nothing inherent in the concept of "Small Schools" that says that the schools actually have to be small organizationally. Consider the problems cited at Lebanon.

But first, why was change even considered?

> Lebanon was dogged for years by a worrisome dropout rate.

This is a serious problem, at least for educators. However, in Lebanon one has to consider the community, a community that may not value education beyond its ability to produce farmers and labor for the local market and for whom the dropout rate was really not that important. Consider:

> [In Lebanon] where grandfathers and fathers once went to work in the mills straight out of high school, there was outrage that the high school's new incarnation was more focused on collegiate prep classes, and less on vocational education.

The groundwork had simply not been done to determine whether change was even appropriate for this community. This is the first principle when viewing a problem systemically—have all factors that affect the outcome of a plan been considered? If there is a basic resistance to the change, then it is not worth the financial expenditure to force the change.

ATIS is founded on the concept of *affect relations*. Affect relations determine the structure of the system which in turn determines its outcomes; that is, the *predictions* of what will happen as a result of these affect relations. Even intuitively, it should be clear that where there are *resisting affect relations* that these will make it difficult if not impossible to carry out objectives which these affect relations oppose. *Affect relations for change* must be weighed against *resisting affect relations to change* to determine the *strength* of each and which will dominate the system. This can be done even without a rigorous logico-mathematical analysis of the system parameters.

For any school system, and, in particular, for school systems in small communities, it should be clear that the affect relations established by the parents and even the students of the community will have a substantial effect on the ability to induce change in that community. Where those influences are contrary to the planned change, the likelihood of success in carrying out the planned change is greatly compromised.

Unfortunately, the desire to "forge ahead" will meet with less-than-expected results.

> Even the biggest local boosters of small schools, like Lebanon Superintendent Jim Robinson, concede that some changes [from those planned] will need to be made, like restoring some vocational emphasis. Still, Robinson said Lebanon will forge ahead, even without the Gates grant.

Unfortunately, pursuant to *ATIS*, making “some changes” will not solve the problem since the problem is systemic and any *change* changes the system. If the impact of a change on the entire system is not considered, then the actual outcome will be distinctly different from what is intended. What could have been done may have been relatively simple and would have cost far less in initial capital outlay. Once again, let’s get back to see what the problems are as stated by the residents of Lebanon.

> In Lebanon, scheduling problems abounded, frustrating teachers who were already skeptical after years of seeing new fads in education tried, then discarded. Test scores and other academic indicators haven't budged, and there were widespread fears that the school was fragmenting too fast, a particularly touchy subject in a smaller city where community is forged on the football field and at graduation ceremonies.

The most significant thing about the above observations is that it, once again, indicates the lack of basic support for the change. Teachers were already skeptical, thus not completely supporting the change. Hence, the *resisting affect relations to change* compromised the efforts for change.

Systemic change requires initial system-wide support of the change. This is as much of a sales and promotional effort as an organizational effort. However, the financial outlay should not be made until it is clear that the basic support has been obtained to make the venture a success.

The “scheduling problems,” however, should not have been a problem. There is nothing easier to do than schedule classes when appropriate software is available and the school system is structured to assure desired scheduling outcomes. This is accomplished by implementing the *large-school structure with a small-school feel*. And this solves the student problems also.

Forging community unity on the football field and at graduation ceremonies? The “problem” answers itself—all four “schools” function as one when it comes to football games and graduation ceremonies. This has been the solution in many larger cities with “Charter Schools” that do not support athletics and other extracurricular programs. Quite simply, the students go to where the athletics are offered and all graduate in one ceremony. This seems to be a “problem” where there is none.

> Students were upset at being separated from friends who had been assigned to other "learning academies", annoyed that they could no longer arrange to be in a particular teacher's classroom, and worried that their choice of courses had been narrowed.

> "It really ticked me off," said Dallas Oeder, 16. "I couldn't take all the classes I wanted to."

There is no separation from friends when all students remain in the same building, and where the "learning academies" are with respect to curriculum structure only. There is nothing in a large school organization that precludes students from being with the same teachers throughout their high school career. And, if a student from outside their clique decides to take a course offered only by a certain "academy," then it is an easy scheduling problem to put the student in the class for that year, or half year, depending on the desired course. Further, under such a structure, there is no narrowing of available courses. And, no student should ever have to say: "*I couldn't take all the classes I wanted to*." Never should school structure preclude a student's education.

As seen from the lessons in Lebanon, Oregon, the solution to expenditure of funds for the outstanding results of the "Small School" movement is to make certain that a proper *systemic analysis* of the school system is initiated first. *ATIS* and its applications can predict the expected outcomes of a reorganization **before** large amounts of capital are expended.

Further, *ATIS* and its applications can help to assure the support of the local community before any reorganization is implemented by focusing on those groups who have to be convinced of the viability of the reorganization in order to assure its success.

When moving into small communities steeped in tradition and education expectations, it is recommended that the desired approach should be: *large-school structure with a small-school feel*.

ATIS and its applications can assist in this analysis and design, and predict reorganization outcomes **prior** to the expenditure of large amounts of capital.

For the Educologist, we will now consider how to use *ATIS* directly for analyzing a school system; and then, what must be done to directly develop education theories. In order to directly develop education theories we must determine how to convert hypotheses to axioms so that *ATIS* can be utilized to analyze a specific school system that will address specific questions concerning that school system.

Background Reports and Developing Education Theory

However, to get a better understanding of how *ATIS* was developed and what it can do, please see the background and development of *General Systems Theory* from which *ATIS* is derived, and to see the technical background upon which *ATIS* is based, by going to the reports at the following links.

General Systems Theory (GST): **GST Background Summary / Critical Developments for a Logico-Mathematical Theory / A Purposeful Existence and Operation Implies Predictability / Intentional Systems Theory**

Developing an *ATIS-Topology:* **Topology and Intentional Systems Theory / Properties of Topological Spaces for an Intentional Systems Theory / Topological Vector Fields / Constructive Development of a Topology for an Intentional System**

ATIS Theory Development: **ATIS Properties (Basic Properties / System / General System / Affect Relations / Transition Functions / Descriptive Analysis of General System / Affect Relation Properties) / [Graph-Theoretic Connected Properties / Information Theoretic Properties]**

It is recommended that as many of the above reports be read as is possible and necessary before proceeding.

> There is a long history in the social sciences of researchers attempting to develop a consistent and comprehensive theory of education, theory of learning, theory of behavior, and other theories of concern to social scientists, but to no avail. And, even with this failure, they persist with the following lament by Ary, D., Jacobs, L. C., & Razavich, A. (1985, p. 19):
>
> > In spite of their use of the scientific approach and accumulation of a large quantity of reliable knowledge, education and the other social sciences have not attained the scientific status typical of the natural sciences. The social sciences have not been able to establish generalizations equivalent to the theories of the natural sciences in scope of explanatory power or in capability to yield precise predictions.

What they fail to recognize is the reason for this lack of theory development. It would seem that with over 100 years of failure, one would begin to look for the reasons that the achievements of the physical sciences have not been attained in the social sciences. How many more decades, scores, or centuries is it going to take before educologists and social science researchers realize that an "*accumulation of a large quantity of reliable knowledge*" does not result in theory?

What we will herein determine is that first, the "*scientific approach*" is not even used by physical scientists, regardless of their claims to the contrary, and second, an "*accumulation of a large quantity of reliable knowledge*" does not result in theory.

What we will do is present an empirical theory for intentional systems that is applicable to all of the social sciences, and that the theory must be developed as an axiomatic theory.

In order to get the most out of this report, certain background is necessary. This report is presented both in a descriptive manner as well as mathematical. But for either approach, there is one work that should be read before proceeding too far with this report. That work is the seminal work prepared and published by Steiner, E. (1988) on theory construction: *Methodology of Theory Building*. The research provided therein is critical reading for any serious reader of this study.

Further, a background or at least a basic knowledge of logic and mathematical logic, in particular, is necessary to understand some of the more technical work in this report, although one can proceed without it and those that are interested in such background can go to the links provided for such background. However, the theory can be comprehended even without that mathematical background.

This report should be of value to anyone interested in the social sciences and, in particular, one who is attempting to develop a theory within the social sciences.

In particular, it will be seen that hypothesis-driven research can never result in theory. This is something that is consistently missed by researchers in the social sciences. We will here see that theory can never be obtained as the result of hypothesis-driven research, which to date is essentially all research in the social sciences and, hence, the inability for any theory development.

For background that may be necessary for an understanding of this research, please go to the following links:

What is ATIS?: **What is an "Axiomatic Formal Empirical Theory"? / What is a "Theory Model"? / What is an "Intentional System"?**

Hypothesis-Based Research Methodologies:

Theories of Learning: **Current Theories of Learning—A Review of the**

Literature

Theory Construction in the Social Sciences: **A Challenge to Learning Theorists / Levels of Theory Construction**

Axiomatic Logics for ATIS: **The Argument for a Symbolic Logic / Intentional and Complex Systems / Axiomatic Temporal Implication Logic / Symbolic Logic / The *ATIS* Sentential Calculus / Modes Ponens / Modus Talens / Axiomatic Sentential Calculus / List of Logical Schemas / Significance of SCTs (*System Construction Theorems*) / Logical Schemas / The *ATIS* Predicate Calculus / Theory Building / Class Calculus / Relation Calculus / *ATIS* Calculus / *ATIS* Options Set Defined / Definitions of Logical Operations in Proofs**

A note needs to be made concerning the definition of 'theory' in the social sciences.

The definition of 'theory' in the social sciences has actually been well-defined in much the same way that is intended in this report. For example, Donald A. Shutt, Jr., Professor at the University of Wisconsin, in an online pdf file, http://www.ssc.wisc.edu/~jpiliavi/357/theory.white.pdf, states that 'theory' is defined as follows:

> I. What is a theory?
>
> A. [A theory is a] logically interrelated set of propositions about empirical reality. These propositions are comprised of:
>
> 1. Definitions: Sentences introducing terms that refer to the basic concepts of the theory
> 2. Functional relationships: Sentences that relate the basic concepts to each other. Within these we have
>   a. Assumptions or axioms
>   b. Deductions or hypotheses
> 3. Operational definitions: Sentences that relate some theoretical statement to a set of possible observations
>
> B. Why should we care? What do theories do?
>
> 1. Help us classify things: entities, processes, and causal relationships
> 2. Help us understand how and why already observed regularities occur
> 3. Help us predict as yet unobserved relationships
> 4. Guide research in useful directions
> 5. Serve as a basis for action. "There is nothing so practical as a good theory."

From the Office of Behavioral and Social Sciences Research, New England Research Institutes, an online company, https://www.neriscience.com/, we also have 'theory' in the social sciences defined as:

> A theory is a set of interrelated concepts, definitions, and propositions that

explain or predict events or situations by specifying relations among variables.

Whereas in this report there may be some variations in terminology, the above definitions essentially state what 'theory' is whether in the social sciences or the physical sciences. And, as essentially stated above concerning what theories do for us, in this report it will be seen what the purpose of a theory is:

> ***The purpose of a theory is to provide the means to develop mathematical, analytical, or descriptive models that predict counterintuitive, non-obvious, unseen, or difficult-to-obtain outcomes***.

Whereas in the social sciences, the "mathematical" development of a theory is normally restricted to statistical measures, in this report we focus on a theory being axiomatic. The notion of theory has not changed, just the type of mathematics used to define the theory. In all fields it is contended that a theory should be "analytical", although the type of analysis may be different. In the social sciences, descriptive models are the norm when applying a theory to empirical observations. In this report, we argue for more logico-mathematical applications.

After developing the type of theory to be utilized, it is then seen that the purpose is to be able to "***predict counterintuitive, non-obvious, unseen, or difficult-to-obtain outcomes***." The point here is simply that if an outcome is already known or easily discernable, then there is no need for the theory. A theory may be able to be developed, but for what purpose if the outcome is already known? If you already know that a student will not learn mathematics if the student is not taught mathematics, then what is the purpose of designing a theory that predicts just such an outcome?

Hypothesis-Based vs. Axiom-Based Research Methodologies

The "Hypothesis" and "Axiom" Distinction

Social scientists rely on hypothesis-based research and attempt to use that methodology to develop social theories. To understand the fallacy of such an approach, the distinction between *hypothesis* and *axiom* must be understood.

In the social sciences, *axioms* and *hypotheses* are frequently considered to have the same meaning. However, in theory development, these two terms are distinctly different.

In fact, the distinctions between *axiom* and *hypothesis* provide strong confirmation why there has been no comprehensive theory developed for the social sciences, and why hypothesis-driven research cannot provide a basis for any such theory development.

Essentially, the distinction is that a *hypothesis* is a conjecture about an observation or a perceived empirical event that is stated as a conclusion of fact that is to be *validated* by experimental testing. An *axiom*, on the other hand, is a statement that relates properties of a theory, or the components (objects) of a theory to its properties. An *axiom* is theory-based; a *hypothesis* is empirical-based.

When a hypothesis is stated, there is no intent that it is meant to develop theory—it is meant to be validated as an assertion of fact. Social scientists then attempt, after-the-fact to utilize the results of the testing of the hypothesis to somehow develop theory. However, since the validation of the hypothesis is nothing more than that, there is nothing by which the hypothesis can be related to other properties or hypotheses that could result in theory development.

Very simply, hypotheses are not designed to develop theory.

The Writing of a Hypothesis as Opposed to an Axiom

Consider the following statement:

HYPOTHESIS: Student choice and independence are the primary motivators for learning.

As stated, this is a hypothesis. It is stated as a conclusion of fact that is to be validated. If it is validated, it provides no relevant relation to any other statements that might be part of a theory and there are no leading assertions from which additional theory statements can be derived. This is so even if the statement is framed as an implication as follows:

HYPOTHESIS: If student choice and independence are related to learning, then establishing student choice and independence in the classroom will confirm them as the primary motivators for learning.

Now consider the following statement:

AXIOM: Students are independent systems (where *independent system* is a property defined by a systems theory).

This statement is an axiom as it relates the components of a theory, *students*, to a property of the theory, *independent system*. It is not a conclusion of fact, as there is nothing to validate, but a theoretical construct, an axiom for a theory, that informs us about a theory property by which students are identified—that is, it is assumed that *students* are *independent systems*. Further, this statement is neither "true" nor "false"—it is simply an assertion that is assumed to be *valid*. Further, there is no amount of testing that can ever confirm the validity of this assertion since there is nothing to validate. Whether or not this axiom is *valid* is not the issue, since it is *assumed* that it is. The issue is whether or not the theorems (or *logically-derived, theory-based hypotheses*) that are deductively obtained from it and other axioms of the theory are validated. Validation of theorems (*logically-derived, theory-based hypotheses*) derived from this axiom provides the on-going "preponderance of evidence" that the axiom is a warranted valid assertion within the theory.

An additional point needs to be made about this axiom. It may be contended, as is often done in the social sciences, that if it is found that there is a student who is not an "independent system," then the axiom is demonstrated to be "false" and the entire theory must be discarded. To the contrary, such an approach is applicable to hypotheses but not to axioms. If an empirical example is found that refutes a hypothesis, then the research is complete and the hypothesis is rejected. However, an axiom is a basic assumption, not a hypothesis. If a student is found not to be an "independent system," as *independence* is defined within the theory, then that student simply is not considered with respect to the theory—the criteria for the basic assumption has not been met. A comparable example from geometry would be where an artist considers parallel lines to meet at "infinity" whereas in Euclidean geometry they do not. Very simply, the one geometry does not refute the other; they are simply two different geometries. If a student is not an "independent system," then that student is not part of the class of students that are being analyzed with respect to the theory that contains the above axiom. When it is asserted that there is nothing to validate with respect to the axiom that is exactly what is meant—the axiom is assumed to be valid and all analyses proceed upon that assumption. If the empirical evidence demonstrates that a particular event does not meet the criteria for the axiom, then all that means is that that event is not analyzed with respect to the theory being considered. This should be a welcome outcome for all educologists who believe that students should be treated "individually"—they in fact are members of distinct systems that require distinct theories! [Such distinct theories are provided for by the *Options Set* of *ATIS*.]

Now, if as a result of the definition of *independent system* along with other system properties and axioms it is determined that "*individual choice and independence are motivators for learning*," then it is as a result of some theory derivation, and not an *a priori* assertion of fact. Then, through various empirical analyses it can be determined whether or not in fact "choice and independence" are the "*primary* motivators for learning." However, even the validation of this conclusion, should it actually be derived, will depend on all of the assumptions and qualifications it took to arrive at this conclusion. Tests are not set up at the discretion and "creativity" of a researcher, but are determined by the parameters of the theory.

When designing tests for hypotheses, the burden is on the researcher to determine if all parameters have been accounted for; hence, the frequent occurrences where two evaluations of the "same" hypothesis results in different outcomes. With an axiomatic theory that results in the derivation of a theorem, it is the axioms and theorems that dictate the parameters of the experiment. Have all assumptions of the theory been accounted for in the design of the experiment?

The distinction between *axiom* and *hypothesis* is seen to be quite profound for theory development, and it is important to keep clear their differences.

Examples of Hypotheses in the Social Sciences and Converting Them to Axioms

Cognitive Load Theory

To see the distinction between hypotheses and axioms and how the latter may lead to theory, consider the following hypothesis taken from the social sciences as propounded by Tracey Clarke, Paul Ayres, and John Sweller (Clarke, 2005):

HYPOTHESIS: Students with a low-level knowledge of spreadsheets learn mathematics more effectively if the relevant spreadsheet skills are learned prior to attempting the mathematical tasks.

The results of testing supported this hypothesis. For our purposes the greater concern is how this hypothesis was developed and whether it may lead to theory development. The rationale for the hypothesis is stated by the researchers as follows:

> According to cognitive load theory, instruction needs to be designed in a manner that facilitates the acquisition of knowledge in long-term memory while reducing unnecessary demands on working memory. When technology is used to deliver instruction, the sequence in which students learn to use the technology and learn the relevant subject matter may have cognitive load implications, and should interact with their prior knowledge levels. An experiment, using spreadsheets to assist student learning of mathematics, indicated that for students with little knowledge of spreadsheets, sequential instruction on spreadsheets followed by mathematics instruction was superior to a concurrent presentation. These results are explained in terms of cognitive load theory. (p. 15)

The process by which this hypothesis was developed is a retroductive process; that is, cognitive load theory was used as a model to develop assertions about learning mathematics. However, there is confusion concerning this process since it is claimed: "*These results are explained in terms of cognitive load theory*" (see last sentence from above quotation).

If in fact the results "are explained in terms of cognitive load theory," and *Cognitive Load Theory* (CLT) is in fact a theory, then this hypothesis is a theorem of CLT and should be deductively obtained from that theory as a theorem to be validated; or, possibly, it is an interpretation of CLT and was derived as an abductive process—that is CLT was used as a model of mathematics learning and the content of the desired hypothesis was substituted for the content of CLT.

> As we will see later when considering the distinction between *retroduction* and *abduction*, *retroduction* is a process of "moving backward", whereas *abduction* is a process of "taking from".

However, it is not claimed that either approach was used, so the question moves to whether or not CLT is actually a theory, or is it a hypothesis that has been validated through various tests? If it is a theory, then we may be able to determine in what sense it is claimed that CLT "explains" the hypothesis.

First, the use of the term *theory* in the context of CLT indicates that it is not either a formal theory or an axiomatic theory. If it is a theory, then it appears to be a descriptive theory. But, even as a descriptive theory, it appears to be very limited in scope and functions more like a hypothesis since deductive derivations are difficult to obtain. But, let us look at this more carefully.

Cognitive Load Theory, developed by J. Sweller, is founded on the following four principles:

- Working memory, or short-term memory, has a maximum capacity identified as *maximum cognitive load*
- Information that exceeds *maximum cognitive load* is lost
- Learning requires that *cognitive load* remain below some value that is less than *maximum cognitive load*
- Long-term memory is consciously processed through working memory

NOTE: The "theory" and "axioms" presented below are for the sole purpose of demonstrating a possible construction of an axiomatic theory and in no way is to be construed as a replacement for the *Cognitive Load Theory* developed by J. Sweller. In fact, it is only as a result of the careful development of CLT that it is possible to derive an axiomatic theory therefrom. In general, it is very difficult to ascertain axioms from a descriptive theory, since they frequently are so vaguely worded that explicit statements of their assumptions are difficult or impossible to determine. Fortunately, CLT is not such a theory.

Provided below is a preliminary development for a *Theory of Memory and Learning* that is retroductively-derived from CLT. Briefly presented are the primitive terms, initial axioms, definitions and a few theorems of the theory.

Theory of Memory and Learning

PRIMITIVE TERMS: *Cognition*, *memory*, *working memory*, *consciously*, *cognitive load, mental structures, patterns,* and *languages*

AXIOM 1: Working memory is that memory which is used to consciously process information.

AXIOM 2: Working memory has a maximum capacity identified as *maximum cognitive load*.

AXIOM 3: Working memory that is maintained below *maximum cognitive load* results in short-term memory acquisition.

AXIOM 4: Long-term memory is consciously processed through working memory.

AXIOM 5: Cognition is determined by a sequence of recognizable patterns or languages.

AXIOM 6: Long-term memory is short-term memory that is processed and related to an existing or newly developed cognitive schema, or structure.

DEFINITION 1: **'**Cognitive schemas' are memory constructs that map short-term memory cognition onto devised mental structures that interpret immediate cognition.

DEFINITION 2: **'**Learning' is defined as that processed cognitive load that results in the acquisition of short-term memory.

As a result of these axioms and definitions, we obtain the following theorems:

THEOREM 1: Information that exceeds *maximum cognitive load* is not cognizable.

PROOF OF THEOREM 1:

- Working memory has a maximum capacity identified as *maximum cognitive load*. (Axiom 2.)
- That which exceeds maximum capacity is not cognizable. (Definition of 'maximum'.)

Another way of stating Theorem 1 is:

THEOREM 1: Information that exceeds *maximum cognitive load* is lost.

This statement of the theorem is the second statement of the four principles cited above for CLT.

THEOREM 2: For learning to occur, *cognitive load* must remain below *maximum cognitive load*.

PROOF OF THEOREM 2:

- Cognitive load that exceeds maximum cognitive load is not cognizable and, therefore, not processed. (Theorem 1.)
- Working memory that is maintained below *maximum cognitive load* results in short-term memory acquisition. (Axiom 2.)
- Short-term memory acquisition results in learning. (Definition of 'learning'.)

Now the problem is to determine if the hypothesis relating to learning mathematics considered previously can be derived from this theory.

Stating the hypothesis again:

HYPOTHESIS / THEOREM 3: Students with a low-level knowledge of spreadsheets learn mathematics more effectively if the relevant spreadsheet skills are learned prior to attempting the mathematical tasks.

PROOF OF THEOREM 3:

- Students do not have cognitive schemas relating to spreadsheets. (Assumption of Theorem 3.)
- Students do not have cognitive schemas relating to mathematics. (Assumption of Theorem 3.)
- Lack of cognitive schemas precludes long-term memory. (Axiom 6.)
- Spreadsheet cognition precedes mathematics cognition. (By Axiom 5 and assumption of Theorem 3, the spreadsheet structure provides the basic "language" by which mathematics is learned.)
- A spreadsheet cognitive schema must be developed for long-term memory to take place. (Axiom 6.)
- Therefore, relevant spreadsheet skills must be learned prior to the learning of mathematical tasks. (Conclusion of Theorem 3.)

The importance of this axiomatic development is that now a much stronger claim can be made concerning Theorem 3. Whereas the initial researchers could only claim: "*These results are explained in terms of cognitive load theory*," it can now be claimed more strongly: "*These results are deductively obtained from the "Theory of Memory and Learning" and are validated by empirical testing*."

But what is the far-reaching effect of this second approach? By validating Theorem 3 the researchers have not only validated their "hypothesis" (theorem) but have now provided support for the theory. This validation has now initiated a process that, hopefully, will eventually provide a "preponderance of evidence" that the theory consistently provides valid outcomes. By framing CLT as an axiomatic theory, every validation of a theorem (or *logically-derived, theory-based hypothesis* if you want) validates not only the theorem but the theory. Eventually, we will be able to obtain theorems deductively from the theory and proceed with confidence that the outcome is accurate, with or without further validation. This is very important, since otherwise every hypothesis must be continually validated in every new setting, in every new school, in every new learning environment.

Whereas the hypothesis has been validated for this one group of students learning mathematics from a spreadsheet, what can we say if instead of a spreadsheet, new computer software is utilized? Will they have to learn the software before learning the mathematics? At first glance, the answer should be "obvious" even without any testing. But, for the sake of making a point, the point is also "obvious"—we have already provided the proof that they would have to learn the software and we do not have to, once again, conduct testing to validate the theorem.

But now, what about results that are not so obvious?

Theory of Memory and Learning—Applications

Theorem 3 provided content that is not stated in the theory axioms. Applications of this theory are the result of the logical process of abduction; that is, the theory content is determined independent of the theory and substituted for the theoretical constructs. For example, "working memory" of the theory is replaced by "spreadsheet" and "mathematics" by the specific application. Additional theory applications can be obtained by interpreting various cognitive schemas.

For a non-obvious theory outcome, consider the following schemas that have been established as part of long-term memory:

(1) Learned behavior described as the schema "assertive"; and
(2) Learned behavior described as the schema "attention-to-detail."

When students learn to keyboard it is frequently asserted that in order to improve speed and accuracy they must practice keyboarding. However, if that were accurate, then anyone who has been keyboarding for many years should be doing so at approximately 60 words-per-minute with great accuracy—whereas this is not the case. There must be more to developing speed and accuracy than practicing keyboarding. Here it is noted that empirical observation refuted the prevailing hypothesis. The alternative was then not derived from the empirical observation, but from recognition that the *Theory of Memory and Learning* should apply to this empirical event—a retroductive process. From the *Theory of Memory and Learning* it is determined that these students have developed certain cognitive schemas defined as *assertive* and *attention-to-detail* that are independent of content.

As a result of these cognitive schemas and the process of abduction by which theory content is determined, the following theorem is obtained:

THEOREM 4: Keyboarding speed can be improved by any off-task activity that increases one's *assertiveness*; and keyboarding accuracy can be improved by any off-task activity that increases one's *attention-to-detail*.

Theorem 4 is a direct result of Axiom 6 and Axiom 5. Theorem 4 is a non-obvious result of the *Theory of Memory and Learning* that was derived from *Cognitive Load Theory*. While there is anecdotal evidence that Theorem 4 is valid, actual validation or refutation of Theorem 4 is left to those who are more skilled at constructing appropriate tests. Whether Theorem 4 is found to be valid or not, the efficacy of an axiomatic theory has been demonstrated as being one that results in non-obvious conclusions. And, as seen here, <u>an axiomatic theory does **not** have to be formal</u>, although the formalization of this theory may result in conclusions that are even more unexpected.

The practical implications for instruction are quite far-reaching. If a student is having difficulty learning a subject or skill, the student can be helped in such learning, not by simply repeating the subject or skill over-and-over, but by diverting the student's attention to a comparable subject or skill that is of more interest to the student. We have probably all seen the movie Karate Kid in which the student is told to wash and wax a car with certain motions, of course, whether he liked it or not. But then, when he went back to learn his blocking and fighting skills, the same motions were implemented, making him a much better karate practitioner. Sometimes, this may have been called "transfer of learning", but in fact it is much more profound. It is developing desired skills by learning skills that are not obviously relevant. Or, when learning mathematics, students may not actually see the importance of such learning and just refuse to learn. A practical application may be to send the students out with a certain amount of money, real or fake, and told to buy as many things as possible with that money. The student who returns with the most "things" wins! Obviously, more creative "applications" can be developed by a creative teacher. The point is simply that we often use this "transfer-of-learning" technique without realizing just how powerful such learning is. Now CLT has shown that it is extremely powerful and should be a more relevant part of the instructional process.

The next step would be to make the theory more comprehensive. Several theories may be related to CLT such as the *Information Processing Theory* by Miller, G. (Miller, 1956), *Human Memory Theory* by Baddeley, A. D., *Cognitive Principles of Multimedia Learning* by Moreno, R. and Mayer, R. E. (Moreno, 1999), *Parallel Instruction Theory* by Min, R. (Min, 1992), *Anchored Instruction* by Bransford, J. D. (Bransford, 1990), and *Social Development Theory* by Wertsch, J. V. (Wertsch, 1985), among others. All of the relevant theories could be brought under one theory that provides the first principles from which all others are derived. Then, all validations further not only the specific research but provide greater confidence in the theory that is founded on the first principles. Such unification would provide a basis by which the unexpected may actually be determined rather than describing that which is already recognized.

Once again, we have a challenge for Learning Theorists—devise the umbrella theory under which all of the above cited theories can be brought by providing the first principles, basic assumptions upon which all theories rely.

Instruction Theory

Another example for converting a descriptive theory to an axiomatic theory comes from Merrill, M. D. (2002, pp. 43-59).

Merrill presents five hypotheses that he refers to as *first principles* of learning and asserts a premise that these principles "are necessary for effective and efficient instruction" (2002, p. 44). Further:

> If this premise is true, there will be a decrement in learning and performance when a given instructional program or practice violates or fails to implement one or more of these first principles.

He continues:

> These five first principles stated in their most concise form are as follows:
>
> 1. Learning is promoted when learners are engaged in solving real-world problems.
> 2. Learning is promoted when existing knowledge is activated as a foundation for new knowledge.
> 3. Learning is promoted when new knowledge is demonstrated to the learner.
> 4. Learning is promoted when new knowledge is applied by the learner.
> 5. Learning is promoted when new knowledge is integrated into the learner's world.

These first principles are already well-stated as axioms with but minor modifications. In fact, he also has already provided corollaries for these hypotheses. The conversion to axioms is accomplished as follows:

1. If students are engaged in solving real-world problems, then student learning will increase.

2. If a student uses existing knowledge when learning new knowledge, then student learning will increase.

3. If new knowledge is demonstrated for the student, then student learning will increase.

4. If a student applies new knowledge, then student learning will increase.

5. If a student integrates new knowledge with existing student knowledge, then student learning will increase.

Whereas these first principles are easily converted to the form of axioms, these axioms do not provide a basis for a theory. The reason is that they all have the same conclusion, thus not providing any means to relate the axioms. What this then tells us is that these axioms are actually *prescriptions for learning* that have been derived from a more robust theory—a learning theory. The antecedents of the implications provide the *prescriptions for learning* as follows:

- Solve real-world problems
- Use existing knowledge
- Demonstrate new knowledge
- Apply new knowledge
- Integrate new knowledge with existing knowledge

As stated, however, these prescriptions for learning but describe events or empirical observations that can be tested, rather than the foundation for a theory. Rather than an *Instruction Theory*, we have an *Instruction Prescription* that may be of great value to teachers.

If these prescriptions are to be derived from theory, then some or all of these antecedents would have to be the conclusions of other axioms. In fact, these *axioms* would probably be *theorems* within the more robust theory. This then reverts back to the challenge presented earlier:

> Is there a Learning Theorist who can develop the basic principles, the first principles in fact, that will encompass not only the other learning hypotheses cited, but now these principles that seem to have strong support for being valid guidelines that will improve students' learning?

Now we can return to our earlier questions.

What is a 'Theory Model'?

'**Theory model**' will be explicated below in the section on *Theory Development*.

What is an 'Intentional System'?

What is meant by 'Intentional'?

'**Intentional**' is used here to describe a person or group of people who have specific goals which their actions are designed to achieve.

For example, an education system has specific goals, the education of students, that it is designed to achieve.

A military system has specific goals, the protection of a society, that it is designed to achieve.

A person has specific work-goals that he/she "intend" to achieve.

These are all examples of a person or group of people who are *intentional*; that is, they are "goal-oriented".

What is a 'System'?

'**System**' is the connection of objects; for example, people, that are considered to be functioning as a unit.

Each of the above three examples are *systems*—an education system, a military system, and an individual person system.

Formally, 'System', $\mathbb{S}$, is defined as the following ordered-pair:

$\mathbb{S} =_{df} (\mathbb{S}_O, \mathbb{S}_R)$, where $\mathbb{S}_O$ is the *object-set* and $\mathbb{S}_R$ is the *relation-set*.

"$=_{df}$" is read: "is defined as".

However, this definition of 'system' will be refined later in this study.

We also now see that there are additional terms that have been added that we will have to understand if we are to proceed in a clear manner that avoids confusion; for example, 'functioning' and 'unit'. However, such an endeavor will result in an unending sequence of terms or will result in a circularity of our definitions.

As a result, it will be necessary to start with certain undefined terms that it must be presumed all readers of this study will comprehend. And, it must also be presumed that all readers of this study will have a certain basic background in order to fully comprehend this study.

What Basic Background is Required to Understand this Study?

It is presumed that all readers will be able to follow the development of the basic logic that is used; e.g., the *Sentential Calculus* and *Predicate Calculus*, among others.

There are numerous textbooks on these subjects where the basic knowledge can be acquired, although, to a certain extent, a development of the logics will also be included in this study so that all readers will be informed of what is required.

It is not presumed that a reader has a knowledge of axiomatic theories, since that is one goal of this study—to discuss the nature of and value of axiomatic theories, to provide the rationale for axiomatic theories and why they are necessary for any definitive empirical study where individual-predictive behavior is desired. While today, practically all studies in the social sciences, if not all, rely on statistical-based analyses, and such analyses are only, and can only be, group-predictive. If individually-predictive outcomes are desired, then an axiomatic-based analysis **must** be utilized.

Further, it is presumed that any serious reader will obtain and read the seminal work by Steiner, E. (1988): *Methodology of Theory Building*.

Also, the work on APT developed by Frick, T. (1990a), "*Analysis of Patterns in Time (APT)*", will be required as it provides part of the logical basis for *ATIS*:

What Undefined Terms are Required?

The required undefined terms will be presented as necessary during this study. For now, we start with the following undefined terms:

'Intentional', 'Object' (which may also be referred to as 'Element'), and 'Empirical'.

Therefore, what is *ATIS*?

As previously stated:

ATIS is an *axiomatic formal empirical theory*
designed specifically for *intentional systems*.

ATIS is an empirical theory that is designed so that selected parameters can be evaluated to determine projected outcomes in view of these parameters.

It should be noted that *ATIS* does not apply only to education systems, but to any *intentional system*. As noted previously, there are many intentional systems, some of which are: education systems, military systems, terrorist network systems, corporate systems, and individual person systems. It is just that in this research, we will restrict our concern to education or learning systems. A distinction between *education systems* and *learning systems* is that we will consider *education systems* to be the more formalized institutional systems formed as school systems run by a government or private enterprise. Whereas a *learning system* can be any individual learning program normally structured by an individual for limited goals.

Applications of *ATIS*

One of the main features of *ATIS* is that it can be used as the basic logic for computer simulations; for example, *SimEd*, *SimTIE*, as designed and developed by Theodore W. Frick, Associate Professor and Web Designer, Indiana University, and other such programs. One program, in particular, that will utilize *ATIS* is SimTIE (*Simulated Totally-Integrated Education*). Thus, *SimTIE* will have an axiomatic empirical theory for its logic which will provide empirical-based predictive outcomes. That is the advantage of utilizing *ATIS* as the basic logic. The theory on which SimTIE is founded has been developed by Frick, T. (Frick, 2016).

The outcomes of computer simulations, or computer models, are dependent on the program designed to analyze the selected input parameters. There are essentially two types of programs that can be used for computer models—*Scenario-Based* and *Logic-Based*. Most often, and especially for the "Sim" models, a *Scenario-Based Model* is used. Such models are dependent on the imagination of the designer and comprehensiveness of the data included in the program.

Scenario-Based Models

Scenario-Based Models are defined as programs that provide *scripts* to determine outcomes. The scripts can be *narrative* or *quantitative*.

> *Narrative scripts* characterize the *qualitative parameters* of a system; that is, the *social*, *philosophical*, and *individual* parameters and their *uncertainty of future outcomes*. That is, the predictiveness of the model is pre-determined by the programmer.
>
> *Quantitative scripts* define the *scientific facts*, known or credible data, and *quantitative models* that are used to *determine future outcomes*. Again, the predictiveness of the model is pre-determined by the programmer in terms of what scientific facts are included in the model.

However, regardless of the type of script, their content is closed; that is, there are a limited number of possible outcomes, and the scripts predetermine the outcomes. Friedman, T. (April 1999) recognizes this closed characteristic of *Scenario-Based Models* in his report on SimCity, "Semiotics of SimCity," when he states:

> Of course, however much "freedom" computer game designers grant players, any simulation will be rooted in a set of baseline assumptions. SimCity has been criticized from both the left and right for its economic model. It assumes that low taxes will encourage growth while high taxes will hasten recessions. It discourages nuclear power, while rewarding investment in mass transit. And most fundamentally, it rests on the empiricist, technophilic fantasy that the complex dynamics of city development can be abstracted, quantified, simulated, and micromanaged.

Logic-Based Models

On the other hand, *Logic-Based Models* are not dependent on analyses of predetermined values, but on the logic of a theory that has been shown to be valid for the target empirical system; for example, an education system. The theory describes the *empirical system* in terms of its affect relations, properties, and axioms. The theory logic is then used to project outcomes founded on the theory with respect to input parameters. The value of the *Logic-Based Model* is then determined by the skill of the designer to provide an appropriate and accurate program that corresponds to the input. Designed properly, a *Logic-Based Model* should be able to provide projected outcomes that are open-ended.

Unlike *Scenario-Based Models* that are closed due to the limited number of scripts, *Logic-Based Models* have an infinite number of outcomes.

This is especially so with the logic designed for the *ATIS Model*. Due to the number of axioms involved, over 200 with more being added, there are initially tens-of-thousands of theorems that can be obtained. However, the SCTs (*Structural Construction Theorems*) provide for an open-ended and essentially an infinite number of additional theorems. The reason is that new affect relations, properties, or system-descriptive parameters can be inserted into the SCTs that will automatically generate thousands of additional theorems. The *Logic-Based Model* is not dependent on what has been initially programmed for the logic, but what is subsequently programmed as a result of new system parameters.

The strength of a *Logic-Based Model* will be seen in what follows, and additional analyses relating to the two types of models will be discussed.

Theory Development

Elizabeth Steiner has explicated the concepts of 'theory' and 'model' in great detail. She has also explicated the relation between 'theory' and 'model'. We will start with her analyses (1988). For a more historical development of her work, see the following upon which her 1988 work is founded: (Maccia & Maccia, 1966), (Maccia, 1965), Maccia, 1964a), (Maccia, 1964b), (Maccia, 1962a), (Maccia, 1962b), and (Maccia & Maccia, 1962c).

It is commonly believed that developing theory is derived from one of two logical processes—*induction* or *deduction*. Theory development, however, is not derived from either.

Induction brings together many observations from which it is claimed a generalization, or pattern, can be obtained. Such, however, is not the case.

It is clear that in order to obtain the pattern, the observer must be able to recognize the pattern, and therefore has brought the generalization to the data. The data does not induce the generalization, the observer does.

This is a mode of inquiry, but it is not theorizing. Today induction provides the basis for the data-mining technologies that are widely used to develop structure from unstructured data. The structuring of unstructured data is not theorizing. Induction is a means of evaluating data so as to recognize and develop patterns. The patterns, when logically organized, are the theory that the data confirms by induction.

Deduction is a means of explicating existing theory; that is, "to clarify and complete theory" (Maccia, 1962b). The theory presents the postulates or axioms upon which the theory is based. Deduction then provides the logical process by which the theory is made explicit through its deductive statements. Deduction generates conclusions of the theory in the form of theorems or hypotheses that are to be evaluated for validity.

Deduction explicates a theory into statements, and *induction* evaluates the statements.

What then is the logical process by which theory is developed? *Retroduction*.

Retroduction

'*Retroduction'* is a "moving backward." From one perspective we move backward to devise another perspective.

For example, the "*Holographic Paradigm*" may provide a perspective for education. Considering that a holograph can be generated from any of its facets, it may suggest that a student may learn, not by focusing on the subject of concern directly, but by developing coordinated skills in a discipline not normally considered as being directly relevant. But, the "parts" of the divergent discipline may have "facets" that in fact produce the entire "hologram" in the subject of concern. For example, one may learn how to interpret historical events by taking acting lessons whereby the skill is developed that allows one to become immersed in a period thinking and lifestyle.

In terms of theory development, one theory that can be used as a model for developing another theory is a "devising theory."

Retroduction is the process of using one theory as a model to devise another theory. Therefore:

(1) *Retroduction* devises theory,

(2) *Deduction* explicates theory, and

(3) *Induction* evaluates theory.

'Retroduction' and 'Abduction' Confusion

There is a prevailing misconception concerning '*retroduction'* and '*abduction'*.

Retroduction is normally, if not universally, defined as *abduction*. Such a definition is in error. First, there is a recognizable distinction between a "taking from," *abduction*, and a "moving backward," *retroduction*.

It is presumed that Peirce generally defines 'abduction' and 'retroduction' as the same, although a careful reading indicates that he does not. An analysis of this confusion is worth considering due to its almost universal acceptance.

The *'retroduction'* and *'abduction'* confusion seems to have come from the work edited by Hartshorne, C. and Weiss, P., *Collected Papers of Charles Sanders Peirce*, (Hartshorne & Weiss, 1960). For example, if one goes to the index for Volume 1, the reference for 'retroduction' is: "*see* Abduction." The implication is that they are the same. But, when we look at the first reference for 'abduction', §65, we find that the two are not the same at all. Peirce writes:

> **§10. KINDS OF REASONING**
>
> 65. There are in science three fundamentally different kinds of reasoning, Deduction (called by Aristotle συναγωγή or αναγωγή), Induction (Aristotle's and Plato's ἑπαγωγή) and Retroduction (Aristotle's ἁπαγωγή), but misunderstood because of corrupt text, and as misunderstood usually translated *abduction*. Besides these three, Analogy (Aristotle's παραδειγμα) combines the characters of Induction and Retroduction.

It should be clear that 'retroduction' and 'abduction' are not the same, and that they have been equated only because of "corrupted text."

So, what is the distinction between 'retroduction' and 'abduction'? Consider the following examples.

> A *direct affect relation and its measure* in a behavioral topological space are defined in terms of a *mathematical vector*. That is, it is recognized that the concept of "vector" is applicable to this behavioral theory. This transition was recognized as a result of affect relations being interpreted as "force fields."
>
> Gravitational and electromagnetic force fields are vector fields; fluid velocity vectors, whether in the ocean or the atmosphere, are vector fields; and weather pressure gradients are vector fields. Affect relations within a behavioral system are vector fields—they are dynamic. They exhibit both direction and magnitude. They exhibit the change and flow of any other empirical vector field.

This process of applying an interpretation to the mathematical construct "vector," is a logical process of *abduction*. This is not a process of "moving backward," but a process of "taking from." The mathematical measure is simply being applied to the content of a behavioral theory.

There is no theory development; there is simply an interpretation of the theory by mathematical means. The mathematical concept of a vector field is utilized as a measure to further explicate the theory. The affect relation concept was already in the theory, so it is clear that no theory development was accomplished. Was there a retroduction of the "form" as a single predicate from mathematics? No. What is being utilized here is simply the definition of a vector field. The definition of "vector field" is being "taken from" mathematics in order to deductively explicate the theory of affect relations. As Thompson remarked following the classification of the SIGGS properties by Frick: "*It was recognized that the Structural Properties represented the topology of the theory*." Such recognition was a deductive process and not a retroductive process. The mathematical vector field theory was not used to devise the SIGGS theory; it was used to explicate the SIGGS theory. (*SIGGS* is an acronym for: **S**et theory, **I**nformation theory, **G**raph theory, and **G**eneral **S**ystems Theory.)

This study defines 'retroduction' and 'abduction' as distinct logical processes, and such that they complement the logical process of 'deduction,' as follows:

- ***Deduction*** is the logical process by which a conclusion is obtained as the implication of assumptions. That is *deduction* explicates theory.

- ***Retroduction*** is the logical process by which a point of view is utilized to devise a conjecture or theory. That is, *retroduction* devises theory.

- ***Abduction*** is the logical process by which a theoretical construct of one theory is utilized to analyze or interpret the parameters of another theory. That is, *abduction* interprets theory.

- ***Induction*** is the logical process by which the conclusions of a theory; that is, the theorems or hypotheses, are empirically evaluated for validity. That is, *induction* evaluates theory.

These distinctions are formalized below. While the proofs of the following theorems are beyond what is required to understand *ATIS*, they are provided since they introduce concepts that may be of value to some researchers; e.g., various logical concepts, as well as new concepts like isostruct and isosubstantive.

While the *Deduction Theorem* is a standard part of mathematic logic, this study extends this analysis to include the *Retroduction Theorem* and *Abduction Theorem*. The proof of the *Deduction Theorem* can be found in many mathematic texts on advanced mathematical logic.

Deduction Theorem

The *Deduction Theorem* will be stated first. The applicable logical schema of the Sentential Calculus is:

If $\mathcal{P} \supset \mathcal{Q}$, then $\mathcal{P} \vdash \mathcal{Q}$; and If $\mathcal{P} \vdash \mathcal{Q}$, then $\mathcal{P} \supset \mathcal{Q} \mathbin{..\equiv..} \mathcal{P} \vdash \mathcal{Q} \mathbin{.\equiv.} \vdash \mathcal{P} \supset \mathcal{Q}$

Where "⊃" is read "implies", "⊢" is read "yields", and "≡" is read "if and only if" and means an equivalence. The periods, ".", before and after the equivalences indicate the grouping priorities. If parentheses are used, the above statement would be shown as follows:

If $\mathcal{P} \supset \mathcal{Q}$, then $\mathcal{P} \vdash \mathcal{Q}$; and If $\mathcal{P} \vdash \mathcal{Q}$, then $(\mathcal{P} \supset \mathcal{Q} \equiv (\mathcal{P} \vdash \mathcal{Q} \equiv \vdash \mathcal{P} \supset \mathcal{Q}))$

The *Deduction Theorem* is a statement of the following implication:

$$\mathcal{P} \vdash \mathcal{Q} \mathbin{.\supset.} \vdash \mathcal{P} \supset \mathcal{Q}$$

The statement of the *Retroduction Theorem* and *Abduction Theorem* are much more complex.

Retroduction Theorem

We will first take a look at the concept of *Retroduction* as defined by Steiner:

> Given theories $\mathcal{A}$ and $\mathcal{B}$, theory $\mathcal{A}$ is a devising model for theory $\mathcal{B}$ if there is a subset, $\mathcal{C}$, of $\mathcal{A}$ such that the predicates of $\mathcal{B}$ are a representation in substance or form of the predicates of $\mathcal{C}$; whatever is true of $\mathcal{C}$ is true of $\mathcal{B}$; and not whatever is true of $\mathcal{B}$ is true of $\mathcal{A}$.

Initially it would appear that the following implication holds: $\mathcal{A} \supset \mathcal{B}$. However, as Steiner points out, "The theory or conjecture that emerges (conclusion) contains more than the theory or point of view from which it emerges (premises). The implication, then, can only hold from the conclusion to the premise"; that is, $\mathcal{B} \supset \mathcal{A}$. It could be argued that the sentential and predicate logic do not hold in this instance. But, if not, we are left with a state of confusion when we are attempting to develop a scientific theory that relies on just such logics. Therefore, it must be assumed that the logic holds and we need to take a closer look at just what is required.

Taking retroduction, as it is conceptually defined; we have that Theory $\mathcal{A}$ is a devising model for Theory $\mathcal{B}$. By this is meant that the predicates for Theory $\mathcal{B}$ are derived as representations from a subset, $\mathcal{C}$, of the predicates of Theory $\mathcal{A}$; that is, $\mathcal{P}(\hat{h})\in\mathcal{B} \supset \mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A}$.

This implication is read: If $\mathcal{P}(\hat{h})$ is a predicate of $\mathcal{B}$, then $\mathcal{P}(h)$ is a predicate of $\mathcal{C}$, which is a subset of $\mathcal{A}$; where the predicates $\mathcal{P}(\hat{h})$ are derived from the predicates $\mathcal{P}(h)$.

But also we have that Theory $\mathcal{B}$ results in more than what was in Theory $\mathcal{A}$; since, otherwise, it would not be an emendation of Theory $\mathcal{A}$, but simply a replication. This emendation of Theory $\mathcal{A}$; i.e., Theory $\mathcal{B}$, that results in more than what is in $\mathcal{A}$ is formally defined as:

$$\exists\mathcal{P}(\hat{h})\in\mathcal{B}[\forall\mathcal{P}(h)\in\mathcal{A}[\sim(\mathcal{P}(\hat{h})\in\mathcal{B} \stackrel{?}{=} \mathcal{P}(h)\in\mathcal{A}];$$

where '$\exists$' is read "there exists", '$\sim$' is read "not" or "it is not the case that", '$\forall$' is read "for all", and '$\stackrel{?}{=}$' is read "is isostruct to"; and isostructism is a mapping of one entity to another to which it is isomorphic or isosubstantive. (That is, "There exists a Predicate $\hat{h}$, an element of $\mathcal{B}$, such that, it is not the case that the Predicate $\hat{h}$ is an element of $\mathcal{B}$ is isostruct to Predicate h an element of $\mathcal{A}$.) This meets the final requirement by Steiner.

The formal definition is read: "There exist predicates, $\mathcal{P}(\hat{h})$, that are elements of $\mathcal{B}$, such that, for all predicates, $\mathcal{P}(h)$, that are elements of $\mathcal{A}$, it is not the case that the predicates, $\mathcal{P}(\hat{h})$, that are elements of $\mathcal{B}$, are isostruct to the predicates, $\mathcal{P}(h)$, that are elements of $\mathcal{A}$.

Further, it is required that the "predicates of $\mathcal{B}$ are a representation in substance or form of the predicates of $\mathcal{C}$." This requirement is formalized as follows:

$$P(h)\in\mathcal{C} \stackrel{?}{=} P(\hat{h})\in\mathcal{B} =_{df} P(h)\in\mathcal{C} \cong P(\hat{h})\in\mathcal{B} :\vee: P(h)\in\mathcal{C} \approxeq P(\hat{h})\in\mathcal{B};$$

where, '$\cong$' $=_{df}$ "is isomorphic to" and '$\approxeq$' $=_{df}$ "is isosubstantive to".

Isostruct $=_{df}$ A mapping of one entity to another to which it is isomorphic or isosubstantive.

Isomorphism =df A mapping of one entity into another having the same elemental structure, whereby the behaviors of the two entities are identically describable by their affect relations.

Isosubstantive =df A mapping of one entity into another having similar predicate descriptors.

Therefore, all of Steiner's stipulations have been met.

As a result, the *Retroduction Theorem* is formalized as follows:

$$\exists\mathcal{P}(\hat{h})\in\mathcal{B}[\forall\mathcal{P}(h)\in\mathcal{A}[\sim(\mathcal{P}(\hat{h})\in\mathcal{B} \overset{?}{=} \mathcal{P}(h)\in\mathcal{A}],$$
$$\mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A} \overset{?}{=} \mathcal{P}(\hat{h})\in\mathcal{B},$$
$$\exists\mathcal{P}(\hat{h})\in\mathcal{B}[\sim(\mathcal{P}(\hat{h})\in\mathcal{B} \overset{?}{=} \mathcal{P}(h)\in\mathcal{A})] \vdash \mathcal{P}(\hat{h})\in\mathcal{B} \supset \mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A}.$$

Proof of Retroduction Theorem

For the purposes of this proof, since the conclusion is simply the result of the assumptions by definition, all that needs to be argued is that the *Predicate Calculus* applies to theory $\mathcal{B}$. To apply, theories $\mathcal{A}$ and $\mathcal{B}$ must be isostruct with respect to $\mathcal{C}$. By assumption, they are. All we have to show is that $\mathcal{P}(\hat{h})\in\mathcal{B}$ represents a consistent set of predicates that have been derived from Theory $\mathcal{A}$ and that they make $\mathcal{B}$ a theory. The formal proof is:

$\mathcal{P}(h)\in\mathcal{A}$	Assumption
$\mathcal{P}(\hat{h})\in\mathcal{B} \supset \mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A}$	Assumption
$\exists\mathcal{P}(\hat{h})\in\mathcal{B}\ \forall\mathcal{P}(h)\in A[\sim(\mathcal{P}(\hat{h}) \supset \mathcal{P}(h))]$	Assumption
$\mathcal{P}(\hat{h})\in\mathcal{B}$ are derived from $\mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A}$	Assumption

All we now need to demonstrate is that $\mathcal{P}(\hat{h})\in\mathcal{B}$ is a consistent theory.

If $\{(\hat{w},\hat{y})\in\mathcal{B}\times\mathcal{B} \mid \mathcal{P}(\hat{w},\hat{y})\} \cong \{(w,y)\in\mathcal{A}\times\mathcal{A} \mid \mathcal{P}(w,y)\}$, then all of the consistent logical conclusions relating to $\mathcal{P}(w,y)$ also apply to $\mathcal{P}(\hat{w},\hat{y})$, by substitution.

If $\mathcal{P}(\hat{h}) \cong \mathcal{P}(h)$, then any component of $\mathcal{A}$ that satisfies $\mathcal{P}(h)$ has a corresponding component in $\mathcal{B}$ that satisfies $\mathcal{P}(\hat{h})$.

Therefore, the components, relations, and predicates which are valid for Theory $\mathcal{A}$ have corresponding components, relations, and predicates in $\mathcal{B}$, resulting in the consistency of $\mathcal{B}$. By definition, the predicates of $\mathcal{B}$ comprise a theory.

Since the theories are isostruct, any proof in $\mathcal{C}$ is applicable to a corresponding proof in $\mathcal{B}$, since they will have corresponding axioms and assumptions. Further, any predicate in $\mathcal{B}$ not in $\mathcal{A}$ can be taken as an assumption or axiom from which resulting theorems can be derived by the *Sentential* and *Predicate Calculi*.

The value of this theorem is that it establishes that the logic of the *Axiomatic Sentential* and *Predicate Calculi* apply to theory $\mathcal{B}$.

As has been shown above, there is a distinction between *retroduction* and *abduction*. The *Abduction Theorem* is given below.

Abduction Theorem. Given theories $\mathcal{A}$ and $\mathcal{B}$, theory $\mathcal{A}$ is a formal model-of theory $\mathcal{B}$ if there is a subset, $\mathcal{C}$, of $\mathcal{A}$ such that the predicates of $\mathcal{B}$ are an equivalent representation in form of the predicates of $\mathcal{C}$; whatever is true of $\mathcal{C}$ is true of $\mathcal{B}$; and whatever is true of $\mathcal{B}$ is true of $\mathcal{C}$.

The formal statement of the Abduction Theorem is:

$$h \equiv \hat{h},\ \mathcal{P}(h) \approxeq \mathcal{P}(\hat{h}),\ \mathcal{P}(h) \approxeq \mathcal{P}(\hat{h}) \vdash \mathcal{P}(h) \in \mathcal{C} \subset \mathcal{A} :\equiv: \mathcal{P}(\hat{h}) \in \mathcal{G} \subset \mathcal{B}$$

Proof of Abduction Theorem:

(1) $h \equiv \hat{h}$ — Assumption

(2) $\mathcal{P}(h) \approxeq \mathcal{P}(\hat{h})$ — Assumption

(3) $\mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A}$ — Assumption

(4) $\mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{G} \subset \mathcal{B}$ — Substitution, 1 in 3

(5) $\therefore \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \vdash \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{G}$ — From 3 and 4

(6) $\vdash \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \supset \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{G}$ — Deduction Theorem on 5

(7) $\therefore \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{B} \vdash \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A}$ — From 4 and 3

(8) $\vdash \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{B} \supset \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A}$ — Deduction Theorem on 7

(9) $\mathcal{P}(h) \approxeq \mathcal{P}(\hat{h}) \vdash \mathcal{P}(h_1), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A} :\equiv: \mathcal{P}(\hat{h}_1), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{B}$ — Definition, 4 & 7 / Q.E.D.

The significance of this theorem is that formal predicates of a given theory that are isomorphic to formal predicates of another theory, define the properties of the second theory.

Types of Models

Steiner presents the concept of 'model' as a dichotomy: '*model-of*' and '*model-for*'.

Intuitively, '*model-of*' corresponds to the familiar type of construction models—model cars, model planes, etc. Also, intuitively, '*model-for*' corresponds to the familiar type of exemplary models—professional models who exemplify appearances or role models who exemplify behaviors.

From these examples, it is seen that a '*model-of*' is a representation of an object, possibly a "scale model"; and a '*model-for*' is the object that is being represented in an ideal; for example, a "super model."

Model-of is a scaled version of the intended object.

Model-for is a paradigm that can be used to describe ideal structures.

These models are designated, respectively, *first-order model* and *second-order model*.

Models that are used to devise theory by retroduction are *second-order models*. They provide the perspective desired to develop the new theory. Thus Steiner defines the relation between the types of logic and the types of models as follows:

Retroductive Logic $=_{df}$ Devising of theory from a *second-order model*.

Deductive Logic $=_{df}$ Explicating a theory for clarification or completeness.

Inductive Logic $=_{df}$ Evaluating a theory to delineate the range of defined objects.

A theory may be further delineated by the referents of the theory. If the theory is about actually existing objects, then it will be called an '*empirical theory*'. For example, theorizing about social referents is an attempt to characterize actually existing objects falling within the domain of some social context or process. Education theorizing is such a theory; it is empirical theorizing. In an *empirical theory*, the statements not only express the nature of the objects, but also the way in which the objects are interrelated.

In view of the preceding, in this study, *ATIS* is a *model-for* education theorizing. *SimEd* or *SimTIE*, on the other hand, are *models-of* education systems.

Now that the type of theorizing has been established, the *ATIS* model will be further explicated. *ATIS* is a logico-mathematical model; that is, it is a *formal model* with logical and mathematical formalizations.

ATIS as a Mathematical Model Theory

In mathematics, model theory is defined as a branch of logic that studies mathematical structure, and, in particular, the structures of axiomatic set theory. *ATIS* is a generalization of mathematical model theory.

Axiomatic set theory is set theory founded on axioms with no empirical content. As set theory is closely associated with mathematical logic, there is an integration of the *Sentential* and *Predicate Calculi* in *ATIS* that results in a formal theory that provides the rigor of deduction and proof.

While the properties and axioms of *ATIS* are initially framed in the context of an empirical theory, those properties and axioms are transformed into a formal logico-mathematical theory that allows for the analysis of *ATIS* as a formal theory.

Mesarović's General Systems Theory Mathematical Model

Others have developed mathematical models for general systems theory. One in particular, Mihajlo D. Mesarović, has developed this area extensively.

Mesarović, M. D., (1972, p. 251) has developed measures for system properties. In his work, Mesarović (1972, p. 264) restricts the measures to "General Systems Theory of Hierarchical Systems." The mathematical measures developed by Thompson in this study are a generalization of the Mesarović measures as extended by Lin, Y. (1999).

However, Mesarović also introduces a "coordination strategy" that will not be applied to *ATIS* measures. This strategy was designed by Mesarović to "adjust" the theoretical projections with actual observations. As described, it appears to simply classify two sets of systems, those that can be "adjusted" and those that cannot. Such a dichotomy is not appropriate for the type of systems here being considered. For *ATIS*, the criteria for verification are with respect to the theorems of the theory without adjustment.

The distinction between the Mesarović approach and that proposed here is that Mesarović relies on models that are "scientifically" developed yet closed, whereas the approach here is founded on the logic of system's theory; such logic resulting in an open-ended theory that provides for an infinite number of outcomes.

As described earlier, the proposed model for this research can be tailored to the specific needs of an empirical system without having to modify the initial program, as would have to be done in a *Scenario-Based Model*.

While Mesarović has contributed greatly to the mathematical development of general systems theory, his system models do not have a basis founded on theory. One such model is WIM (World Integrated Model) that was developed with 49 subroutines. It was quite refined in that it utilized about 21,000 numbers to describe the state of the global system at any one time. An overview of WIM can be viewed online at http://genie.cwru.edu/scenarioanalysis.htm. That overview is provided here:

World Integrated Model (WIM)
DOS/ Windows & Mac versions, coded in FORTRAN and consists of 49 subroutines.
WIM began in 1972 as a joint project between Mihajlo Mesarović and Eduard Pestel. WIM is a model which utilizes 21,000 numbers to describe the state of the global system at any given point in time. The world is divided into 12 regions and represents integrated global variables such as population, energy, natural resources, trade, etc. GLOBESIGHT has superseded WIM. WIM was originally funded by the Volkswagen Foundation.
Contact: Peter Brecke, School of International Affairs,
Georgia Institute of Technology, Atlanta, GA 30318-0610
Email: peter.brecke@inta.gatech.edu
Areas included in model:
Economics, Environment, Energy, Population, Natural Resources

The direction being taken by the study herein presented is distinct from that of most, if not all, other social models—this *SimEd Model* will rely on a *Logic-Based Model* for its projections. It is believed that the parameters are too numerous and the possible outcomes are so extensive that anything less would result in a model that could end up with the same shortcomings as that recognized by Friedman concerning SimCity.

With this introduction to *ATIS*, it is seen that in order to have a legitimate empirical model that can result in empirically-verifiable results, the model must be founded on a Logic-Based Model, and that *ATIS*, being an axiomatic theory, provides just such a basis.

We will now take a more in-depth look at the development of *ATIS* and its value for empirical studies of intentional systems.

'*Theory'* and '*General System*' Definitions

The definitions of *theory* and *General System* must be considered in more detail, and how they can address the needs of those industries concerned with predicting system outcomes founded on various structural scenarios desired by an organization or as the result of empirical observations.

Since the systems of concern are characterized by vast amounts of information, we can come to an understanding of such systems by seeing what is being done in industries that have dealt with large amounts of information. One such industry that must interpret vast amounts of data is the counter-terrorist industry. This industry previously utilized the *Total Information Awareness (TIA) System*, but it was discontinued due to privacy concerns. However, since it utilized data-mining technologies, it would have been limited in its ability to actually provide individually-predictive outcomes.

A report concerning the *TIA System*, however, does highlight the direction being pursued in this industry to respond to the terrorist threat, and provides key points that should be considered when attempting to find a means by which the terrorist threat can be identified.

In a report by the *Defense Advanced Research Projects Agency's Information Awareness Office*, it states:

> "It is difficult to counter the threat that terrorists pose. Currently, terrorists are able to move freely throughout the world, to hide when necessary, to find unpunished sponsorship and support, to operate in small, independent cells, and to strike infrequently, exploiting weapons of mass effects and media response to influence governments. This low-intensity/low-density form of warfare has an information signature, albeit not one that our intelligence infrastructure and other government agencies are optimized to detect. **In all cases, terrorists have left detectable clues that are generally found after an attack**. Even if we could find these clues faster and more easily, our counter-terrorism defenses are spread throughout many different agencies and organizations at the national, state, and local level. **To fight terrorism, we need to create a new intelligence infrastructure** to allow these agencies to share information and collaborate effectively, and **new information technology aimed at exposing terrorists and their activities and support systems**. This is a tremendously difficult problem, because terrorists understand how vulnerable they are and seek to hide their specific plans and capabilities. **The key to fighting terrorism is information**. **Elements of the solution include gathering a much broader array of data than we do currently**, discovering information from elements of the data, **creating models of hypotheses, and analyzing these models in a collaborative environment to determine the most probable current or future scenario**. DARPA has sponsored research in some of these technology areas, but additional research and development is warranted to accelerate, integrate, broaden, and automate current approaches." (*Emphasis added*.)
>
> This reference can be found at the following link for *DARPA Information Office*:
>
> http://spot.pcc.edu/~rwolf/DARPA/darpahome.htm

Therefore, what is needed is a means of identifying the "clues" **prior** to an attack. The solutions are recognized, but the technology required to address these solutions has not been developed. As seen, they are still relying on "*analyzing models in a collaborative environment*". However, analyzing such models can never be predictive, they will always be reactive since the information has to be first obtained and then analyzed. What is needed is a real-time analysis of incoming data that is predictive. What is needed is a "*new information technology aimed at exposing terrorists and their activities and support systems*." As reported herein, *ATIS* is just such a technology.

It is also noted that similar concerns are confronted in our pursuit of a reliable education system where it is desirable to be able to discern outcomes of changes in education methodologies in a timely manner rather than having to wait 12 or more years before results can be recognized.

In response to the terrorist threat, the technologies being sought are characterized as follows:

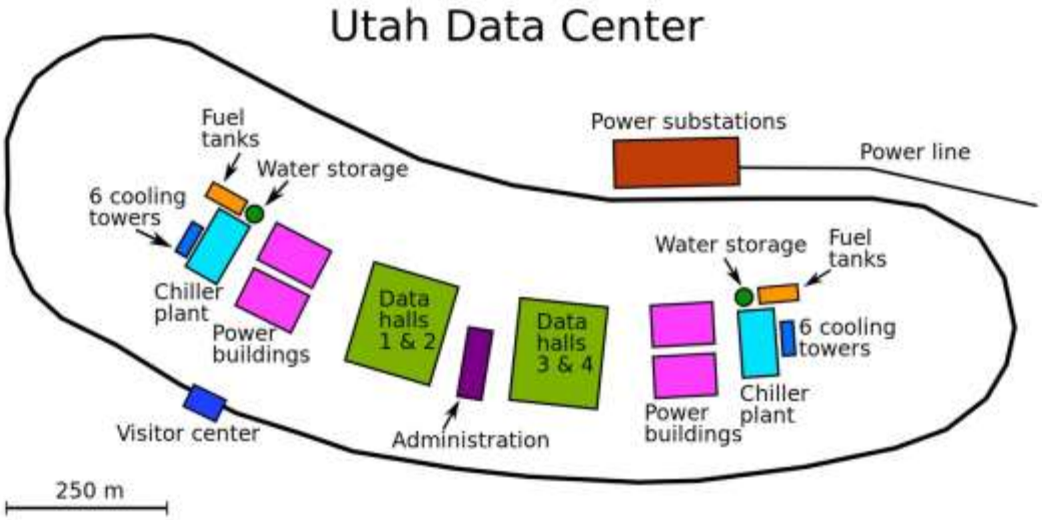
- Real time learning, pattern-matching and anomalous pattern detection
- Human network analysis and behavior model building engines
- Event prediction and capability development model building engines
- Data mining of unstructured data
- Information discovery through statistical methods

As noted, these same concerns can be applied to education systems.

The following charts depict the efforts that the TIA was trying to address, and can be found at the following website:

(http://en.wikipedia.org/wiki/Information_Awareness_Office#Components_of_TIA_projects_that_continue_to_be_developed)

See Diagram 1 below of the Utah Data Center and a description of its purpose.



Utah Data Center

The **Utah Data Center**, also known as the **Intelligence Community Comprehensive National Cybersecurity Initiative Data Center**, is a data storage facility for the United States Intelligence Community that is designed to store data estimated to be on the order of Exabyte's or larger. An Exabyte, EB, is 1 billion gigabytes.

The purpose of the center is to support the Comprehensive National Cybersecurity Initiative (CNCI), though its precise mission is classified. The National Security Agency (NSA) leads operations at the facility as the executive agent for the Director of National Intelligence. It is located at Camp Williams near Bluffdale, Utah, between Utah Lake and Great Salt Lake and was completed in May 2014 at a cost of $1.5 billion.

SOURCE: https://en.wikipedia.org/wiki/Utah_Data_Center

The data center is alleged to be able to process "all forms of communication, including the complete contents of private emails, cell phone calls, and Internet searches, as well as all types of personal data trails—parking receipts, travel itineraries, bookstore purchases, and other digital 'pocket litter'." *Bamford, J. (15 March 2012).* "The NSA Is Building the Country's Biggest Spy Center (Watch What You Say)" *Wired.*

The scope of this project is what is needed as the I_B; i.e. *Information Base* required for large information-based systems. Of course, for education systems, whatever I_B is available and appropriate can be used.

Diagram 1: Diagram of Utah Data Center and Description of Its Purpose.

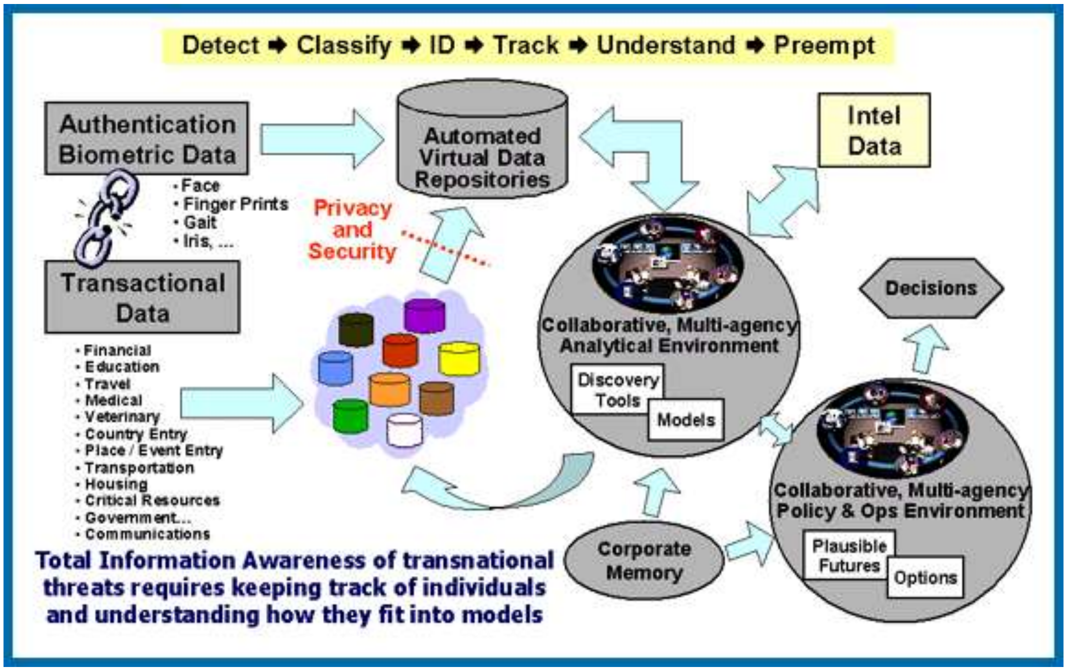


Diagram 2: Diagram of *Total Information Awareness System*, taken from official (decommissioned) Information Awareness Office website

What this chart does show is the vast amount of information that must be integrated and analyzed in order to determine predictive outcomes.

The information required for an education system is shown on the following page.

The diagram has also been expanded for ease of reading.

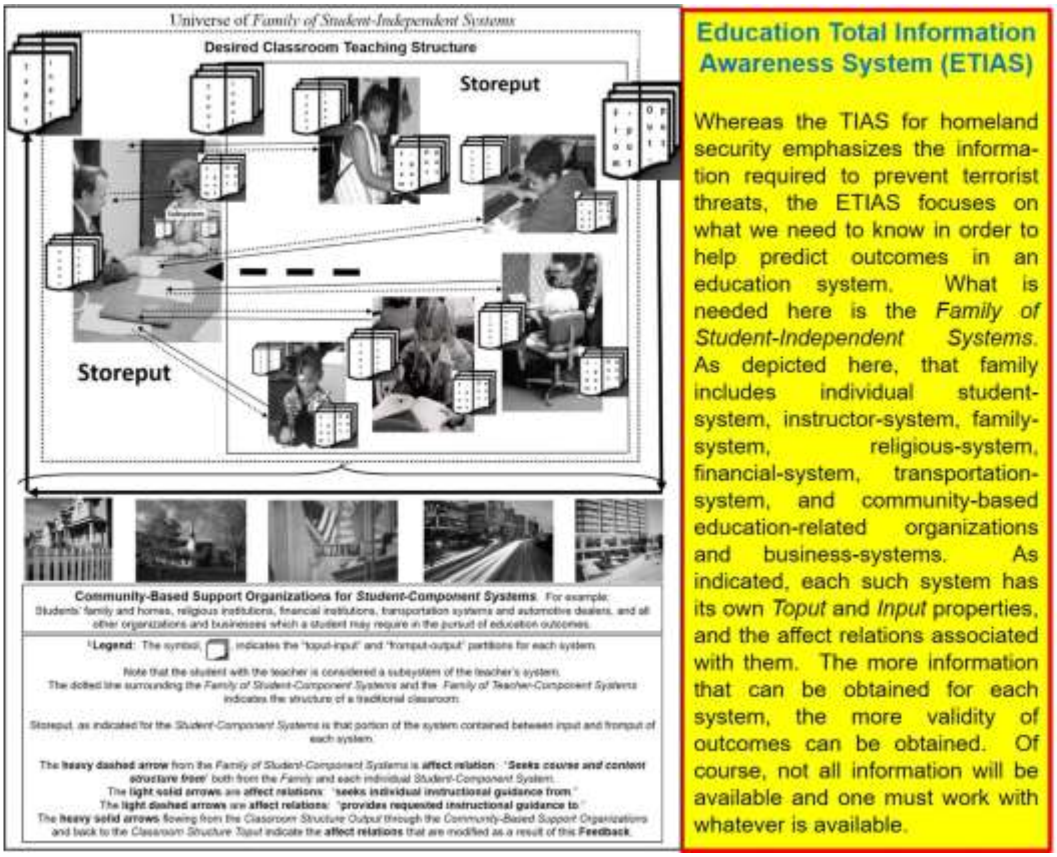
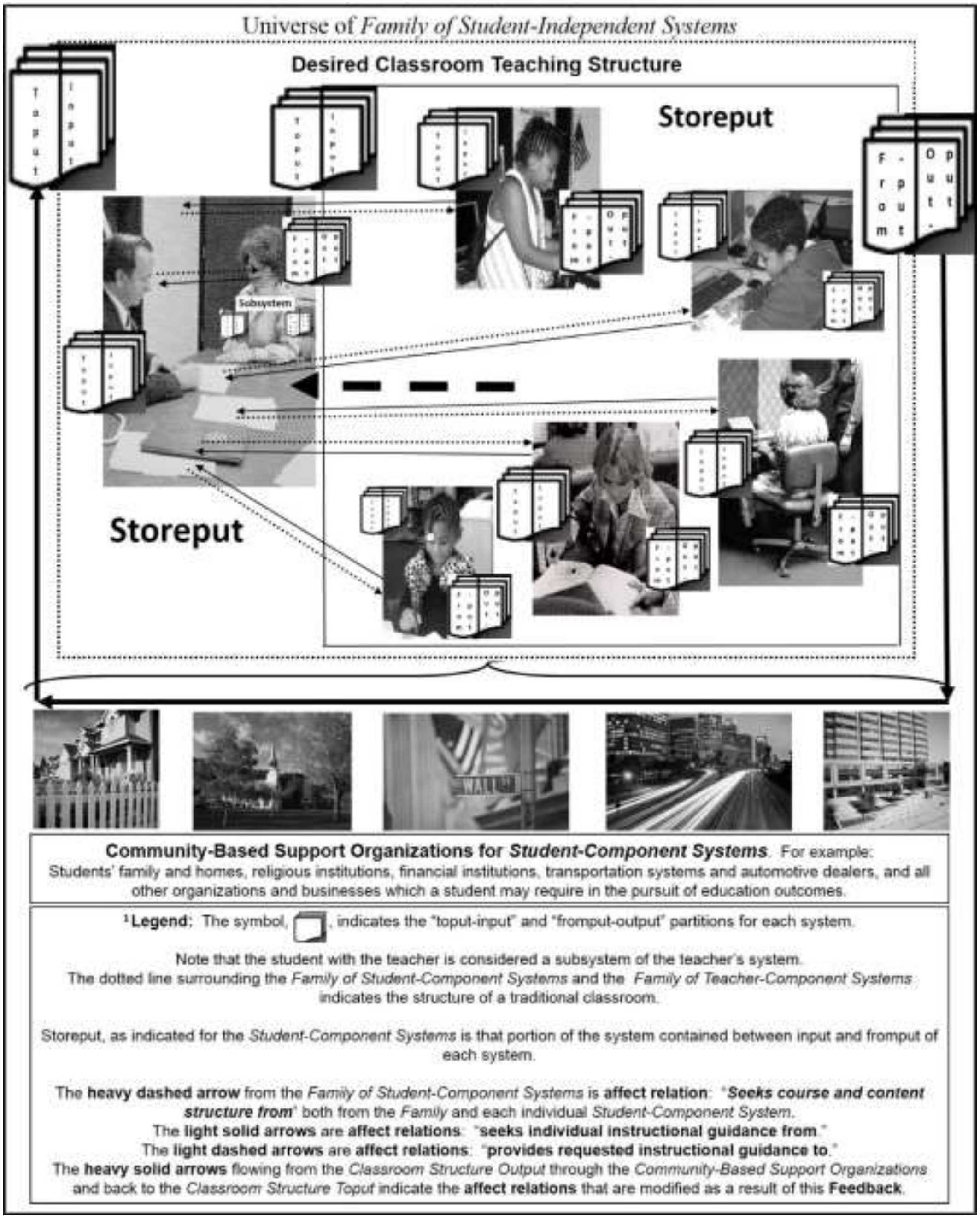


Diagram 3: Diagram of *Education Total Information Awareness System*



For an in-depth analysis of an education system, please refer to the following link: *Theory Development in Education: Implementing the ATIS Options Set*.

However, the difficulty with all of the approaches cited above, the technologies being sought, is that they rely on statistical-based analyses. The same can be said for any industry today where predictive technologies are being considered; and especially in education. What is actually wanted, however, is the ability to detect *discrete indicators*, not *group indicators* that are provided by statistical-based analyses. That is, "**In all cases, terrorists have left detectable clues that are generally found after an attack.**" These are *discrete indicators* that can never be identified by the use of *group indicators*.

For example, in education, we can frequently recognize problems after the fact and can recognize "indicators" that should have pointed us in the right direction. For example, on-going modifications of a school system or a class are done in response to "feedback" that the current plan needs modification. Those indicators are discrete and in order to optimize instruction, they must be found *prior* to the compromising of instruction. Statistical-based analyses, *by design*, **cannot** detect such indicators, since they only "detect" **that which has already happened** and only with respect to the group and not the individual.

When considering how to predict system optimization, we must first understand the nature of the problem and what solutions are even feasible for solving the problem. We will start with the following premise:

Group-Predictive Premise

> Statistical analyses are, by design, only group-predictive, and can—*by design*—**never** be individually predictive; that is, they can never identify *discrete indicators*.

It is important to state this premise since essentially every approach now being taken in education, business, the military, etc. is founded on a statistical-based analysis. Hypotheses are verified by statistical studies. This is especially the case with respect to unstructured data. *Data-mining* is a critical tool for developing ***patterns*** of business or educational behavior. However, such analyses **cannot** provide discrete predictions. Further, in all such analyses, patterns must be determined and that can only be done after the activity is well developed.

An axiomatic theory; however, can provide analyses with respect to discrete indicators and is required for predicting individual outcomes.

We will start by taking another look at the definition of *General System* and how it should be defined in order to analyze intentional systems.

Definition of *General System*

From a review of the literature, it is clear that there are various definitions of *system* as well as *general system*. Some of the definitions are required due to mathematical concerns. Others are very imprecise and are used for descriptive arguments rather than logical or mathematical precision. The definition used here follows the convention in mathematics of a system, $\mathfrak{S}$, being an ordered pair consisting of an object-set, S, and a relation set, $\mathfrak{R}$; that is:

$$\mathfrak{S} = (\mathbf{S},\mathfrak{R}).$$

This definition can be brought into the context of education or terrorist network systems by citing the definition provided by Steiner and Maccia as follows:

System, $\mathfrak{S}$**,** $=_{df}$ A *group* with at least one affect relation that has information (Maccia, E.S. & Maccia, G.S., 1966, p. 44).

$$\mathfrak{S} =_{df} (\mathbf{S},\mathfrak{R}) = (\mathfrak{S}_{\mathfrak{x}},\mathfrak{S}_{\phi})\text{; where } \mathbf{S} = \mathfrak{S}_{\mathfrak{x}} \text{ and } \mathfrak{R} = \mathfrak{S}_{\phi}.$$

> A **system** is an ordered pair defined by an *object-set*, $\mathfrak{S}_{\mathfrak{x}}$, and a *relation-set*, $\mathfrak{S}_{\phi}$.

It is noted that with the development of *ATIS*, the requirement that the affect-relation "has information" has been dropped due to theoretical concerns.

In this study, a more extended definition of system is required to more fully define *General System*. This extension is also required to more clearly define the topology and/or relatedness of a system by its object-sets and relation-sets; as well as allow for a more rigorous and comprehensive development of the system logic required for a logical analysis.

A *General System* is defined within a ***Universe of Discourse***, $\mathcal{U}$, that includes the system and its environment. The only thing that demarcates the systems under consideration is the "Universe of Discourse." And, while that universe may be somewhat fuzzy, whatever systems are being considered will be well defined. In the case of *Education Systems* the boundary of the universe may be quite fluid, or possibly unknown, especially with respect to the object-sets.

$\mathcal{U}$ is partitioned into two disjoint systems, $\mathfrak{S}$ and $\mathfrak{S}'$. Therefore, ***Universe of Discourse*** has the following property:

$$\mathcal{U} = \mathfrak{S} \cup \mathfrak{S}'\text{; such that, } \mathfrak{S} \cap \mathfrak{S}' = \emptyset.$$

The disjoint systems of $\mathscr{U}$, $\mathbb{S}$ and $\mathbb{S}'$, are defined as *system* and *negasystem*, respectively.

System environment and *negasystem environment* are defined as follows:

> ***System environment,*** $\mathbb{S}'$, $=_{df}$ The components of the universe not in the system.
>
> ***Negasystem environment,*** $\mathbb{S}$, $=_{df}$ The components of the universe not in the negasystem.

A ***General System***, $\mathcal{G}$, is frequently defined by the following parameters:

- *Family of Affect Relations Set*, $\mathcal{A}$;
- *Object Partitioning Set*, $\mathcal{P}$;
- *Transition Function Set*, $\mathcal{T}$;
- *Linearly Ordered Time Set*, $\mathscr{T}$; and
- *System State-Transition Function*, σ.

That is: **General system,** $\mathcal{G}$, $=_{df}$ A set of affect relations, partitioned components, transition functions, time set and a system state-transition function.

$$\mathcal{G} =_{df} (\mathcal{A}, \mathcal{P}, \mathcal{T}, \mathscr{T}, \sigma);$$

However, although, this definition of *General System* provides a fairly comprehensive view of what is required to properly analyze the functioning of a system, in order to address the concerns of the desired intentional systems this definition needs to be extended. First, the above cited parameters are defined as follows:

<u>Affect Relation Set</u>: $\mathcal{A}$, the *Affect-Relation Set,* corresponds to the previously stated relation-set, $\mathbb{S}_{\phi}$. Intuitively, this is the set that contains all of the "relations" between elements of $\mathscr{U}$. This is a "set of sets." However, in this case, the sets are defined by specific Affect Relations. For example, these sets will be defined for an education system as follows: *"Teacher-Student Instructional Relation"; "Student-Textbook Relation"; "Student-Parent Relation"; "Administrator-Business Community Relation";* etc. As can be seen, this set can become quite large with numerous subsets, the various relations of an education system. This is where the concern of "system refinement" must be considered; that is, when selecting relations that are to be considered for the education system, the ***<u>Least</u>-Refined Definition Principle*** must be employed.

Least-Refined Definition Principle: Any system can be viewed with greater refinement, but the level of refinement must be minimized to obtain the greatest predictive results—a view that is possibly counter-intuitive.

Possibly the easiest way to visualize this is to remember that you do not want to let the minute details get in the way of being able to see what is going on. If we were cognizant of everything around us, we would not be able to function properly due to all the *noise*.

Object Partitioning Set: $\mathcal{P}$, the *Object Partitioning Set*, corresponds to the previously stated object-set, $\mathbb{S}_x$. Intuitively, this is the set that contains all of the "things" within an education system or terrorist network system or other large system and its negasystem: *students, teachers, administrators, learning materials, community resources*, etc.; or *terrorists, financial resources, supporters*, etc. What is special about this set, however, is that it is a "set of sets"; its elements are subsets of $\mathbb{S}_x$. And, any one object of the system can be in only one subset, hence the name "*Partitioning*." For example, even if a "*student*" at times fills an instructional capacity, the individual can only be placed in one set—either the individual is a "*student*" or a "*teacher*," but not both *at any one time*.

Transition Function Set: $\mathcal{T}$, the *Transition Function Set*, is necessary in order to "move" objects about the Universe, $\mathcal{U}$. The elements of this set are the functions defined by *feedin, feedstore, input-feedintra, storeput-feedintra, feedout, feedthrough, feedenviron,* and *feedback*. Without them, nothing moves. Also this provides for the dynamics of the system whereby individuals, as above, can be placed in the "*student set*" at one time and the "*teacher set*" at another time.

Linearly Ordered Time Set: $\mathscr{T}$, the *Linearly Ordered Time Set*, is required in order to give the intentional systems a *dynamic property*. This set helps to keep the system organized! Intuitively, this set may be the easiest one to apply to the education system. That is, essentially it allows you to attach to an event the appropriate "*time*" that the event occurs. Without this set there would be no *order* or *sequence* to the events of the system. Also, this is necessary for any application of APT Values (Scores) [*Analysis of Patterns in Time Values* (Frick, T., 1990].

System State-Transition Function: σ, the *System State-Transition Function*, is required in order to alter the "*state*" of an education system. Whereas $\mathcal{T}$, the *Transition Function Set*, moves objects about the system, σ changes the state of the system as a result of the new *Affect-Relations* defined by the move or new affect-relations introduced into the system. Both $\mathcal{T}$ and σ produce a change in the system, but each is required in order to define the changed system.

Now that each parameter has been defined and described, let's take another look at the definition of General System:

$$\mathcal{G} =_{df} (\mathcal{A}, \mathcal{P}, \mathcal{T}, \mathscr{T}, \sigma)$$

Here we do need a refinement of the definition to provide greater relatedness of these parameters. That is, an additional parameter must be introduced. The definition is changed as follows:

$$\mathcal{G} =_{df} (\mathcal{A}, \mathcal{P}, \mathcal{Q}, \mathcal{T}, \mathscr{T}, \sigma)$$

In this definition, another parameter has been added—$\mathcal{Q}$ (qualifiers, normally for component-qualifiers). With this modification, we now have the following definition:

__General System Defined:__

> ***General System*** $=_{df}$ a set of affect-relations ($\mathcal{A}$) which determine a set of partitioned components ($\mathcal{P}$) defined by component-qualifiers ($\mathcal{Q}$), a transition functions set ($\mathcal{T}$), a linearly-ordered time set ($\mathscr{T}$), and a state-transition function (σ).

This definition is formalized as follows:

$$\mathcal{G} =_{df} [\mathcal{A} \Vdash (\mathcal{P} (\mathcal{Q}, \mathcal{T}, \mathscr{T}, \sigma))]; \text{ where}$$

> '⊩' is read "determines" or "which determines" or "from which is/are derived", as appropriate for the sentence in which it is used. This symbol is similar in intent to the logical "yields", but whereas "yields" is a logical relation for a deductive proof, this is a predicate relation identifying that which is derived from the existent set.

That is, a system is first recognized by the affect-relations and __**not**__ the components of the system, as is commonly assumed. From the affect-relations, the partitioned components are obtained. Then the component-qualifiers are determined; that is, the properties of the system that determine what components are to be considered members of the system. Then the relatedness of the components through the transition functions is determined, a time assigned, and the state-transition is determined.

Now, let's take a look at the elements of each of these sets.

The sets that define $\mathcal{G}$ have the following elements:

$\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \in \mathcal{A}$;

$T_\wp$, $I_\wp$, $F_\wp$, $O_\wp$, $S_\wp$, $L_\wp$, $\mathbb{S}_{BX}$, $\mathbb{S}'_{BY} \in \mathcal{P}$; that is, *toput, input, fromput, output, storeput, spillput, system logistic qualifier, negasystem logistic qualifier, system background components*, and *negasystem background components*.

$\mathcal{L}$, $\mathcal{L}' \in \mathcal{Q}$; that is, $\mathcal{L}$ are the *system qualifiers*, and $\mathcal{L}'$ are the *negasystem qualifiers*.

$f_{\mathfrak{I}}$, $f_{\mathfrak{O}}$, $f_{\mathfrak{T}}$, $f_{\mathfrak{B}}$, $f_{\mathfrak{E}}$, $f_{\mathfrak{S}} \in \mathcal{T}$; that is, *feedin, feedout, feedthrough, feedback, feedintra*, and *feedstore*.

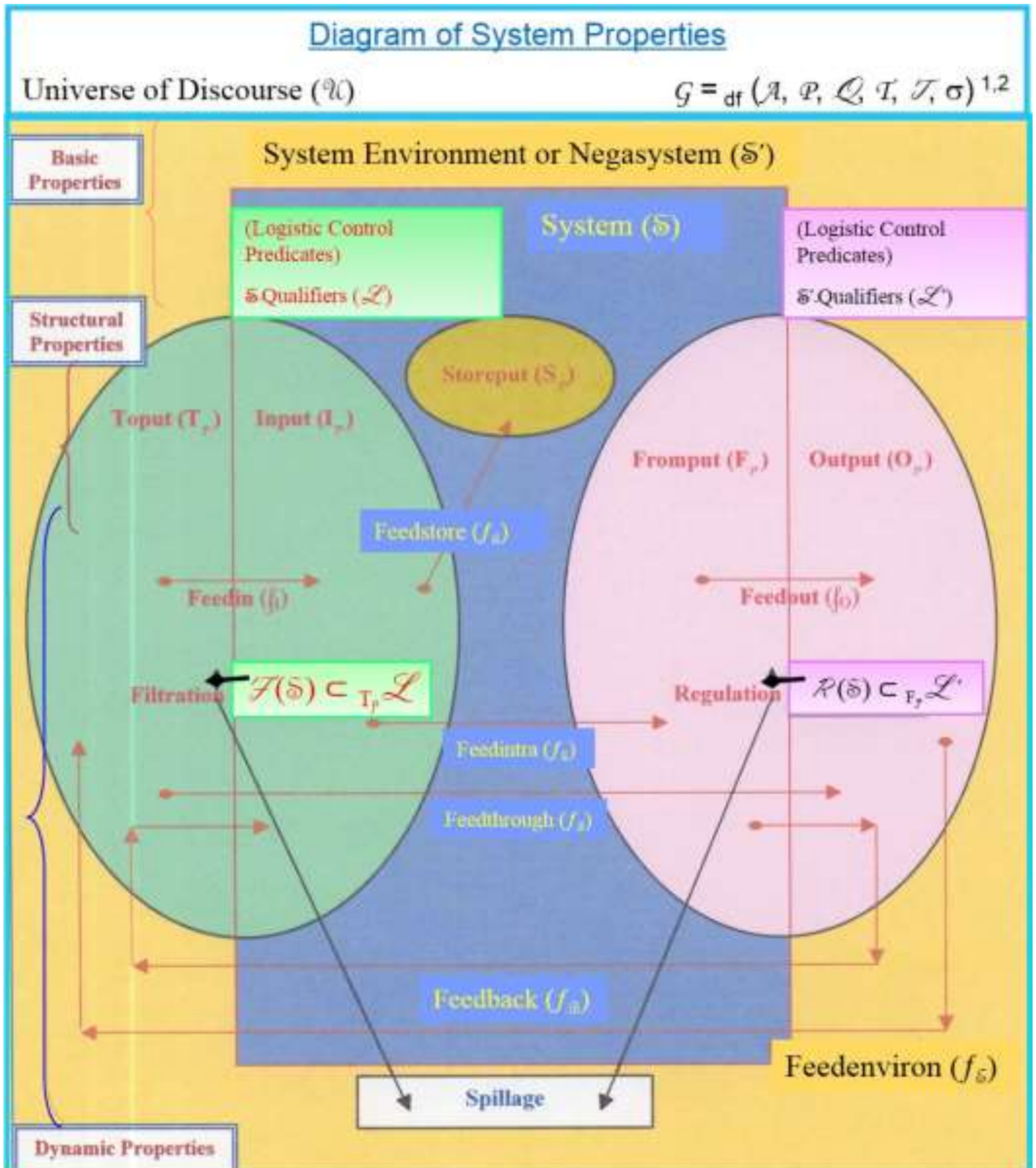
$t_1, t_2, \ldots, t_k \in \mathcal{T}$.

Let the object-set of a *General System*, $\mathcal{G}_O$, be such that $\mathcal{G}_O = \mathbb{S}_X \cup \mathbb{S}'_Y$; where $\mathbb{S}_X$ and $\mathbb{S}'_Y$ are the object-sets of $\mathbb{S}$ and $\mathbb{S}'$, respectively. Then, $\mathcal{G}_O$ is defined by the following:

$$\mathcal{G}_O =_{df} \mathbb{S}_X \cup \mathbb{S}'_Y = (I_\wp \cup F_\wp \cup S_\wp \cup \mathcal{L} \cup \mathbb{S}_{BX}) \cup (T_\wp \cup O_\wp \cup L_\wp \cup \mathcal{L}' \cup \mathbb{S}'_{BY}).$$

Further, as all of these sets are disjoint, the following holds:

$$I_\wp \cap F_\wp \cap S_\wp \cap \mathcal{L} \cap \mathbb{S}_{BX} \cap T_\wp \cap O_\wp \cap L_\wp \cap \mathcal{L}' \cap \mathbb{S}'_{BY} = \emptyset.$$



[1] $\mathcal{G}$ is the *General System*, $\mathcal{A}$ is the *Family of Affect Relations Set*, $\mathcal{P}$ is the *Object Partitioning Set*, $\mathcal{Q}$ is the *Qualifier Set*, $\mathcal{T}$ is a *Transition Functions Set*, $\mathcal{T}$ is a *Linearly-Ordered Time Set*, and σ is the *System State Transition Function*.

[2] $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \in \mathcal{A}$; $T_p, I_p, F_p, O_p, S_p, L_p, \mathbb{S}_{BX}, \mathbb{S}'_{BY} \in \mathcal{P}$; $\mathcal{L}, \mathcal{L}' \in \mathcal{Q}$;
$f_s, f_o, f_E, f_B, f_E, f_s \in \mathcal{T}$, and $t_1, t_2, \ldots, t_k \in \mathcal{T}$.

Diagram 4: Diagram of System Properties

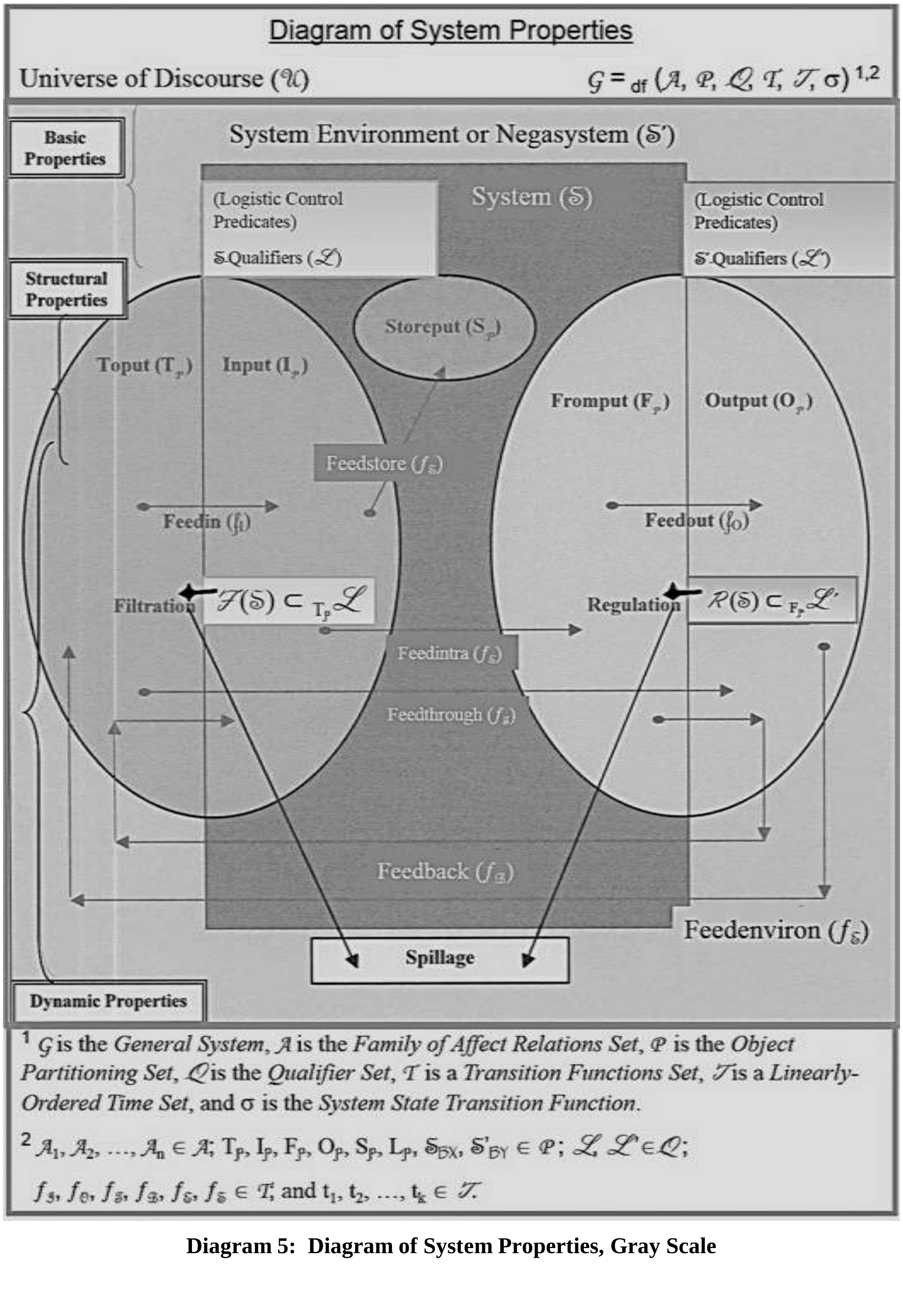


Diagram 5: Diagram of System Properties, Gray Scale

Summary

In this report we have discussed theories and models, and how *ATIS* is developed as an *axiomatic formal empirical theory* designed specifically for *intentional systems*.

Theories of learning were discussed and why it is that they do not and cannot provide a comprehensive and consistent theory, and why hypothesis-based development cannot result in a theory of learning.

We discussed the distinction between scenario-based and logic-based models and why a logic-based model is required if actual predictive outcomes are desired when studying education systems or other types of intentional systems.

We also discussed how an empirical theory must be developed and why a logico-mathematical theory is required in order to gain a real understanding in the social sciences.

A definition of general system was presented and was given an initial formal development.

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